EFFECTS OF CONSISTENCY VARIATION ON NON-NEWTONIAN LUBRICATION

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By

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CERTIFICATE

This is to certify that the matter embodied in the thesis entitled 'EFFECTS OF CONSISTENCY VARIATION ON NON-NEWTONIAN LUBRICATION' by Mr. Samuel Muthiah Athisaya Raj for the award of the degree of Doctor of Philosophy of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under my supervision and guidance. The thesis has, in my opinion, reached the standard fulfilling the requirements of the Ph.D. degree. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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CONTENTS

ABSTRACT

The study of the intervening fluid in the separation gap between bearing surfaces in supporting the imparted load has been a fundamental consideration in the field of Lubrication. The quality, behaviour, range of utility in the physical environment together with the molecular structure of the lubricant and its affinity to the bounding surfaces are important in understanding the lubricating mechanism in the performance of modern machinery.

The need for fairly good theoretical prediction of bearing characteristics is possible when the realistic lubricating conditions such as rheological anomalies or variation of viscosity/consistency in the fluid film, surface roughness, elastic deformation under heavy loads etc. are included in the analysis.

It appears that the properties of the lubricant film in contact with the solid surface are different from that of the bulk lubricant. Kingsbury [1] was one of the earliest investigators to report the rheological abnormalities of the fluid in the proximity of the solid surface. He observed an enhanced viscosity in the region of attraction of surface molecules of the metals. The most important and interesting work in the study of rheological abnormalities of the fluid very near to the bearing surface is the work of Needs [2] who squeezed fluids between two steel plates and observed a stable residual film. More recently, the works of Cameron and his Coworkers [3,4]

indicated an increase in the viscosity with the addition of certain additives and this results in a much larger viscosity than the bulk of the fluid. The presence of additives in the base oil characterises a non-Newtonian behaviour. The non-Newtoni behaviour of the lubricant may be characterised by the power law model.

Another important factor to affect the bearing performance is the thermal effect. This effect is generally studied by coupling Reynolds and energy equations. The complexity of the complete solution to Reynolds equation arises mainly due to the inclusion of energy equation in the calculation procedure. A different approach to avoid energy equation was presented by Tipei and coworkers [5,6] who replaced the viscosity-temperatu relation by a film thickness-viscosity relation. With this approach , an extensive parametric study of lubrication problems especially in the case of power law lubricants may be attempted. This approach may be used to study the consistency variation along the film thickness.

Prasad [7] analysed the lubrication of rollers using a power law fluid considering consistency variation across the film thickness. Some of his results obtained are open to question as the load ratios defined to study consistency variation across the film thickness was considered by taking into account cavitatio and no cavitation load . In the present work a more general model to account for consistency variation across and along the fi thickness is developed to study the effect of this variation on th

lubrication characteristics. Consistency variation across the film thickness is accounted through a consistency ratio factor while its variation along the film thicknessis accounted through a thermal factor. An attempt is made in the present work to use this model for consistency variation in lubricating conditions. The model is used for smooth as well as rough bearings and rigid as well as deforming bearings under heavy loads. The thesis work is divided into seven Chapters.

Chapter 1 is introductory in nature. It contains a selected list of experimental and theoretical results in support of rheological anomalies in the convicinity of the bearing surface. Apart from the natural phenomenon of this rheological behaviour, a brief history of works relating to surface roughness non-Newtonian behaviour of the lubricants, elastohydrodynamics lubrication are dealt with in conformity with the theoretical analysis.

In Chapter 2, the generalized Reynolds equation considering consistency variation across as well as along the film thickness is derived. Subsequently, it has been used to study the effect of consistency variation in the case of rigid rollers. Using Grubin's theory [8] the model for consistency variation is used in the inlet analysis of elastohydrodynamic lubrication. It has been established that the effect of the high consistency peripheral layer is to increase the load capacity in the case of rigid rollers and to increase the minimum film thickness in the ehd lubrication. In the later case , it was theoretically

established that no set pattern of minimum film thickness with respect to the flow behaviour index is observed, When the total peripheral layer thickness is of the same order as that of the minimum nominal film thickness. This is atributed to the complexity of the rheological effects in the ehd lubrication. The effect of the thermal factor in the case of high consistency peripheral layer is to decrease the load capacity in the case of rigid rollers and to decrease the minimum film thickness in the ehd case.

The model for consistency variation is used in the analysis of squeeze film in Chapter 3. It is observed that the presence of high consistency peripheral layer increases the load capacity and response time. The increase is more for pseudoplastics as compared to dilatants. It was concluded that consistency in the peripheral region and surface asperities, it was found that there exists a critical value for the consistency ratio, above which the trend of the load ratios with respect to the flow behaviour index are similar to that for the smooth case. When the consistency is higher in the central layer as compared to that in the peripheral layer, there is an increase in the load capacity for longitudinal roughness. For a high consistency peripheral layer, in the transverse roughness there is greater load capacity and protection against seizure.

In Chapter 6 is studied the squeeze films on rough bearings considering consistency variation. It is concluded that in the

presence of surface asperities, the effect of consistency variation across the film thickness is similar to the case of smooth bearing. The effect of roughness on load and response time is more for dilatants than pseudoplastics.

In Chapter 7 is studied the effect of consistency variation on the inlet pressure for both longitudinal and transverse roughness. It is inferred that an increase in the consistency in the peripheral layer results in an increase in the inlet nominal film thickness. In the case of transverse roughness the inlet pressure increases as the value of the roughness parameter increases for both the cases when the total peripheral layer thickness on both the rolling surfaces is greater or less than the minimum nominal film thickness. The effect of the thermal factor is to decrease the inlet pressure for transverse case and to increase it for longitudinal case.

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CHAPTER 1

GENERAL INTRODUCTION

1.1 EARLY DAYS OF LUBRICATION

The manner in which the intervening fluid in the separation gap between the bearing surfaces supports the imparted load has been the subject of intensive study to understand the lubrication mechanism between the various moving parts of the modern machinery from gigantic structures to minute components of mechanical system. The vital study associated with the lubrication process centres around friction and wear of the machinery during running-in. The inevitable occurance of these natural phenomena cause severe operational disturbances and in some cases lead to total destruction of the entire mechanical system. One of the persistent demands, naturally, is to control friction and wear and mitigate wastage of energy during operation of the system. To meet this demand, the study of the quality, behaviour, range of utility in the physical environment of the lubricating fluid is essential. Further, the molecular structure of the lubricant and its affinity to the bounding surfaces together with the strctural and topographical changes of these surfaces makes the study useful for practical applications.

The lubrication practice is old. During the era of Egyptian Pharaohs, it was found that the axle of the chariot wheel had been lubricated with a sticky and slightly greasy

substance [1]. The substance contained road dirt such as quartz sand and components of aluminium and lime. Its melting point was determined to be 20^{o}F. It was suggested that it could be either mutton or beef tallow, both of which were suitable for axle lubrication in that tropical country. Until very recently the lubricants have been of animal, vegetable or marine origin - mutton tallow, lard, goose grease, castor, cotton seed and other vegetable oils. The use of mineral oils dates back to a little over hundred years. Today straight mineral oils cannot cope up with the situations when the growing demand to transmit greater power through mechanisms smaller in size and weight imposes increasing burden on the operating lubricant. Recently, the synthetic lubricants, with greater thermal and viscosity properties than those of the mineral oils, are used especially in most arduous and transient bearing performances.

Pioneering work in the field of tribology or lubrication was initiated by Adams [2] in 1853, while developing certain patents for railway axle bearings. However, the new science of lubrication on hydrodynamic principles was founded by Petrov [3] who theoretically analysed the friction effects and film lubrication. In the same year Petrov published his theory, Tower [4] demonstrated that, a loaded oil lubricated bearing might be subjected to local

pressure substantially higher than its mean pressure. Three years later, Reynolds [5] independent of Petrov's theory, was able to confirm Tower's results using the properties of the laminar fluid film in the equation which was named after him. Certain solutions to this equation were obtained by him that agreed well with Tower's experimental measurements. Later on, Sommerfeld [6] in 1904 made an elegant theoretical extension to journal bearing lubrication problems.

To obtain solutions to Reynolds equation in lubrication problems, assumptions such as no slip, constancy of viscosity of the lubricant across and along the film thickness etc., are usually made. However, for bearings operating under actual environmental and physical conditions these assumptions may not be valid and hence, the prediction of bearing characteristics based on these theoretical studies is bound to differ with experimental measurements and observations. To predict the bearing characteristics with a fair degree of accuracy close to the actual performance, it is, therefore, indispensable to make generalisations of the Reynolds equation. Accordingly, various generalisations taking into account of the viscosity variation across and along the film thickness, thermal effects, porosity and slip condition, non-Newtonian characteristics of the lubricant, surface roughness, rheological anomalies of the lubricant very near to bearing surfaces, elastic deformation of the bearing suefaces due to a highly

developed pressure etc. are made. Keeping these in view, lubrication characteristics of various designs of hybrid two lobe and three lobe bearings, multilayed porous bearings,etc. apart from the regular geometrical conventional bearings, have been determined. A historical and detailed review of the same would be an Herculean task and beyond the scope of the present work.

1.2 LUBRICATION AND RHEOLOGY

It has been pointed out earlier that the properties of the lubricant film contiguous to the bearing surface are important in determining the bearing characteristics. It appears that the properties of the liquid in contact with a mating solid surface, influenced by the proximity of the liquid to the bearing surface are a subject of intensive study and a matter of controversy. The molecules of the liquid near to adhering surfaces or of the adsorbed layer on the surface are believed to behave differently from the adjacent molecules of the liquid in bulk. The nature of the surface influence and depth of its penetration into the surface led to a variety of divergent opinions. It is generally accepted that the surface zone is not merely a monomolecular layer but a multimolecular layer i.e., the molecular orientation extends effectively to many molecular lengths [7,8]. Adamson [9] pointed out that though evidence of a deep surface orientation was more or less circumstantial, the molecular

orientation does occur probably as a result of perturbations passed on from molecules to molecules.

Several experiments were conducted to examine whether the fluid in the vicinity of the adhering surface developed properties significantly different from that of the fluid in bulk. Kingsbury [10] was one of the earliest investigators to report the rheological abnormalities of the fluid in the proximity of the solid surface. He observed an enhanced viscosity of the fluid in the region of attraction of the surface molecules of the metals. This enhancement might range from a small increase in viscosity to rigidity, and the depth of the affected zone might range from 10^{-7}-10^{-3} mm. The most important and interesting work in the study of rheological abnormalities of the fluid very near to the bearing surfaces is the work of Needs [11] who squeezed fluids between two steel plates and observed a stable residual film. Though some of his observations are questioned, it appears reasonable that an alteration of liquid film occurs near the bounding solid surface which makes it substantially more viscous compared to the bulk of the liquid.

The most telling support to Needs observation is, perhaps, provided by the findings of Fuks [12,13] who conducted a number of experiments using an apparatus similar to that used by Needs. It is of interest to note that an addition of 0.1 % of stearic acid to a chromatographically separated

naphthene-paraffin fraction oil has increased the thickness of the residual film. It is equally interesting to note that lower-molecular weight hydrocarbons such as benzene,hexane, cyclohexane, isooctane and decane exhibited no residual film. It was observed that the film thickness increased with the fatty acid concentration, with surface energy of the solid substrate and with increasing hydrocarbon chain length of both solvent and fatty acid.

The works of Needs and Fuks were a matter of great controversy and criticism by many workers, notably Hayward and Isdale [14]. It has been suggested that the asperities on the solid surface might account for residual film. The dirt and the unsolved material in the fluid film have been also offered as an explanation. Drauglis et.al. [15] observed that if the data of Fuks were to be explained on the basis of dirt, then we had to postulate that that dirt would change its properties with temperature, length of additive molecule and concentration of additives, but such a dirt would be a most unusual material. Derjaguin [16,17] pointed out that the molecular orientation in depths would occur near solid boundaries and give rise to significant rheological changes. Increase in viscosity was observed where traces of polar or surface active substances were present.

More recently,Cameron and his coworkers [18-21] studied the influence of surface active components in various lubrication

systems. The lubricants used were paraffins mainly hexadecane, and the additives consisted of various long chain polymer compounds. It was pointed out that the oil film was much influenced by the presence of additives. The scuffing load was found to increase with the chain length of additive and solvent. Squeeze film studies similar to Fuks were made. It was observed that the plates came to rest at a separation of $2x10^{-4}$ mm. with all fluids including pure cetane. The increase in surface viscosity was greatest when additives were matched with carrier oils in terms of chain length and shape. The results of these studies are explicable by postulating the presence of a surface film 10^{-5}-10^{-3}mm. thick which has a much larger viscosity than the bulk of the fluid.

From these studies we conclude, therefore, that the evidence of the existence of rheological abnormalities (like enhancement of viscosity) in the neighbourhood of solid boundaries on the whole seems to be positive, despite some controversy. It is clear that zones of enhanced viscosity (giving rise to residual film) would have important effects on the lubrication of practical machinery.

As has been pointed out earlier, the presence of additives in base oils gives rise to rheological abnormalities in thin films such as enhancement of viscosity. Further, the incorporation of additives to the base lubricant improves the

primary functions of the lubricant in minimizing sliding friction and offering surface protection against damage and wear [22]. Ordinary mineral lubricating oils contain various polar compounds,many of them are surface active in a non-polar hydrocarbon medium,and thus belong to the class of liquids for which the evidence of the effect of viscosity enhancement is strongest. The rheological anomalies could,therefore, have a considerable effect on the minimum film thickness, which in turn influences wear and fatigue of machine components.

The idea of blending additives to petroleum oils to improve the performance was used in the power, industrial and railway lubricants. Modern lubricant additives are based on many years of scientific research, designed to meet the extreme demands of modern machines and for high performance ratings under actual working conditions. The automative industry and machine equipment manufacturers use additives fortified lubricants to meet severe machine service demands, thereby widening the margin of safety of operations. The additives are incorporated in carrier oils for a variety of purposes. There are additives used as rust inhibitors (Amine phosphates), corrosion inhibitor (sulphurized olefins), fire resistors (hologenated hydrocarbons), detergents (calcium/ barium sulphorates) viscosity improvers (polymethylacrylate, powders of graphite and molybdenum disulphide), extreme pressure (EP) additives (sulphurised fats) etc. There are

certain experimental reports to support that the addition of additives to base oils reduce friction, wear and enhance the lubrication process [23]. In particular, it has been noted that in the lubrication of rollers, the film thickness considerably increases when additives are introduced in the base oils [24]. The additives or surfactants, thus, present in the lubricant enhances the effective viscosity of the lubricant near the contiguous surface which is beneficial to the hydrodynamic lubrication process [25].

The use of EP additives in lubrication is important. It is usually accepted that these additives react with sliding metal surfaces to form a surface film which reduce wear and aids seizure resistance. The reaction between EP additives and sliding surfaces is promoted mainly by the increase in the surface temperature owing to frictional heat [26]. For extreme temperature applications, synthetic liquids such as silicones, esters are used to improve temperature properties. The thickening agents are usually lithium, calcium, sodium, barium and aluminium for temperature stability [27]. A more detailed knowledge than the existing one at present about the mechanism of formation and chemical structure of the reaction layers is of great interest for the development of new and more effective additives [28].

1.3 SURFACE ROUGHNESS

The relation between surface finish and wear is of great practical significance to bearing design engineers. The wear behaviour of the machine surface is affected by surface topography and surface integrity. The surface roughness and the surface waviness are two topographical structures, respectively, describing short range and long range geometric deviations of a surface from the nominal geometric shape [29]. Several attempts have been made to study the effects of roughness following deterministic and stochastic approaches. In deterministic approach, the film thickness is normally assumed to be represented by sine (cosine) function for a given roughness profile and usual hydrodynamic equation is, then, solved to calculate the bearing characteristics [30]. It was showed that load capacity, frictional force etc. are dependent on the amplitude and wave length of these waves representing the roughness profile [31]. In the stochastic approach,the film thickness is assumed to be a stochastic or ergodic function and stochastic average is taken either for the Reynolds equation or for its solution. Chen and Sun [32] pointed out that the method of averaging was practical only for the analytical solution of the Reynolds equation. Tzeng and Saibel [33,34] introduced the stochastic concept of

surface roughness and obtained mean pressures for infinite slider and short journal bearings considering striations running transverse to the direction of motion. Subsequently, Christensen and Tonder [35] derived a generalised form of stochastic Reynolds equation to include surface striations running in the direction of motion as well. A further refinement to reference [35] was made by assuming the flow fluxes to be represented by power series of a stochastic film thickness function. Such a refinement was proposed by Christensen, Shukla and Kumar . Sun [37] studied the effect of two dimensional surface roughness by taking into account the roughness spacing and autocorrelation function describing the roughness structure.

The application of surface roughness theory depends on the characteristic length of the roughness structure. When this length is very much larger than the local film thickness the Reynolds equation is locally applicable ; if this length is too small, Stokes equation is applicable [38]. Sun and Chen [39] and Phan-thein [40] considered stokes roughness. Shukla [41] presented a deterministic approach to study the effects of roughness when the mean height of the surface asperities was of the same order of magnitude as that of the minimum film thickness. He introduced roughness interaction zone along the rough surface together with a purely hydrodynamic zone along

the smooth surface . This approach for roughness was later extended to the case where roughness interaction zones were considered along both the bearing surfaces and a purely hydrodynamic zone in between them [42].

Recently, Patir and Cheng [43,44] presented an average flow model to determine the effects of three dimensional roughness by deriving the average Reynolds equation based on pressure and shear flow factors obtained through flow simulation. Employing this model and using asperity contact pressure obtained through Greenwood and Tipp's model [45], Majumdar and Hamrock [46] studied the effects of roughness on the elastohydrodynamic line contact by calculating the pressure for the entire contact region. Recently, Prakash and Czichos [47] used Patir and Cheng's model for partial elastohydrodynamic lubrication of rough rollers. It has been pointed out that when there is no interaction of surface asperities, it is sufficient to analyse the inlet region; however, in the mixed lubrication regime, it is necessary to analyse the entire contact region. Tonder [48] calculated the average fluid flow and shear stress numerically in the hydrodynamic lubrication of surfaces consisting of adjacent periodic and symmetric unit roughness textures.

Some experimental studies were also reported. Woo and Thomas[49]

presented a review of experimental work relating to the area of real contact, the number of contact spots, the spatial distribution of contact spots and the distribution of their sizes and the relation of all these to roughness and normal load. An interferometric study of the elastohydrodynamic lubricated contact of rough surfaces was reported by Jackson and Cameron [50] and some of Christensen's conclusions on the stochastic model for roughness [51] were also confirmed by it. A technique for measuring the heights of surface asperities in three dimensions has been developed by Tsukada and Sasajima [52]. It was confirmed that three dimensional characteristics of surface asperities on ground, lapped and worn surfaces could be studied by observing three-dimensional representation and contour maps obtained with the measuring system suggested by them.

In the present work, we employ Christensen's stochastic model for roughness to study hydrodynamic and elastohydrodynamic lubrication of certain conventional bearings using power law lubricant.

1.4 ELASTOHYDERODYNAMIC LUBRICATION

As mentioned earlier, under heavily loaded conditions, the bearing surfaces get deformed and the pressure generated in the film is modified due to elastic deformation and viscosity variation of the operating lubricant [53].

The primary consideration in the elastohydrodynamic (ehd) lubrication is the determination of minimum film thickness. Grubin [54] was the first to study the ehd film thickness and obtained a formula for minimum film thickness. Dowson and Higginson [55] modified this formula after solving relevent equations numerically. However, it was pointed out that for all practical purposes these two formulae did not differ too much [56]. Laton on, Christensen [57] modified the film thickness formula for lubrication of rollers. Using Crook's approximation [58,59] and Grubin's theory, many research workers have studied the ehd film thickness. Chow and Cheng [60] studied the effect of roughness with idealised asperities in the ehd contact of rollers. Dyson and Wilson [61] investigated the problem of ehd film thickness of rollers, theoretically and experimentally using power law lubricants. Recently, Sinha et.al [62] analysed ehd lubrication of rough rolling surfaces with/without elastic deformation using power law fluids.

In this work, we employ Grubin's theory and Crook's approximation to study the elastohydrodynamic lubrication of smooth as well as rough rollers.

1.5 NON-NEWTONIAN LUBRICATION

As mentioned earlier, presence of high molecular weight additives (polymers) and dirt particles collected during manufacturing and lubricating processes make the operating lubricant in the bearing clearance deviate from the Newtonian behaviour. Such a lubricant is to be characterized by a non-Newtonian model [63-65]. Hirst and Moore [66] pointed out that many base oils were non-Newtonian under severe operating conditions. Hydrodynamic lubrication of gears and rollers involve rapid changes in the shear rate of lubricant and the relation between shear-stress and shear rate is no longer linear. In analysing the ehd film thickness, Johnson and Cameron [67] have pointed out that the fluids in high pressure ehd contact are essentially non-Newtonian in behaviour. During the last two decades several models such as power law, Bingham plastic Herschel-Bulkey, Reiner-Rivlin, pseudoplastic, viscoelastic etc. [68-74] have been proposed to characterize non-Newtonian fluids.

In recent years, the power alw model has gained attention due to its capacity to characterize many types of lubricants such as polymer solutions and silicone fluids [61]. Ng and Saibel [75] employed the modified model of pseudoplastic power law lubricant in the lubrication of inclined slider bearing and showed that the load capacity was

less than that for Newtonian fluids. Tanner [76] studied the non-Newtonian lubrication of short journal bearing using power law lubricants and showed that a reduction in the coeff-icient of friction might be be obtained with typical power law fluids provided bearing eccentricity was not a limiting factor. Several attempts have been made to study lubrication problemsfor various bearing systems such as squeeze films [77], externally pressurized bearings [78], conical bearings [79], journal bearings [80], roller bearings [64] etc. Recently, Kayer et.al [81,82] studied the behaviour of externally pressurized conical bearings with a power law fluid and pointed out that the load capacity and temperature rise of the fluid increased with the flow behaviour index of the fluid. In this work, the non-Newtonian behaviour of the lubricant is characterized by power law model and a generalised Reynolds equation is derived and applied.

1.6 VISCOSITY/CONSISTENCY VARIATION

It has already been mentioned that, for estimating the bearing characteristics in the actual performance of the bearing system, generalisations to the Reynolds equation to include effects such as viscosity/consistency variation, thermal effects etc. are to be made. Lope [83] modified the Reynolds equation by including viscosity and density

variation along the fluid film thickness. The viscosity variation along the film thickness has been considered by Zinkiewicz [84,85] . Dowson [86] generalized the Reynolds equation by considering the variation of fluid properties across and along the fluid film. Crook [87] assumed that the viscosity varied with pressure and temperature exponentially in the analysis of rollers.

Thermal effects in lubrication have been recognized as an important factor affecting the bearing performances. A complete solution to the Reynolds equation was first obtained by Zienkiewicz by considering temperature variation across the film thickness with temperature dependent viscosity [84].McCallion et. al [88] studied the thermal effects by obtaining a complete solution to momentum and energy equations. Thermal effects are usually considered by assuming the lubricant to be Newtonian. However, as mentioned earlier , due to a variety of causes the lubricant behaviour is essentially non-Newtonian. Hence, there is a need to study the thermal effects in the lubrication situation using non-Newtonian lubricants.

Many workers studied lubrication problems by coupling the Reynolds and energy equations. Rigorous consideration of temperature variation restricts the application of the

solution to a particular set of operational conditions i.e., to a particular set of viscosity temperature relationship and to a specific inlet or initial viscosity. Rigorous variable viscosity analysis is both expensive and limited in application [89]. Further, one of the major difficulties in calculating the temperature of the bearing surface is that the number and size of the areas of contact between bearing surfaces are not usually known. Another difficulty is the experimental one of measuring the temperature at the contact zone in order to verify the proposed theoretical analysis [90].

The complexity of the complete solution to Reynolds equation mainly arises due to the inclusion of energy equation in the calculation procedure. Qvale and Wiltshire [91] eliminated the energy equation from calculation procedure by considering a chosen viscosity profile as an independent variable. This simplifies the calculation substantially and an extensive parametric study of lubrication problems can be attempted. A different approach to avoid the energy equation was presented by Tipei and his coworkers [92-94] who assumed a relationship between viscosity and film thickness under thermal equilibrium. King et. al [95] remarked that a uniform temperature across the fluid film appeared to be reasonable in view of very small

film thickness (.05-1μm) and very high linear velocity (1m/s). Most of the heat generated in the film would be carried away by convection while thermal conduction across the boundaries would play a minor role. Following the approach of Tipei and his coworkers, in this work, thermal effects are considered by replacing consistency-temperature relationship by consistency film thickness relationship by a model akin to one proposed by them.

1.7 IN THIS WORK

In this work, an attempt is made to study the non-Newtonian behaviour of the lubricant with the power law model. While using this model, many research workers derived generalized Reynolds equation without mentioning the domain for which it exists. Further, the non-dimensional scheme adopted by them are subject to controversy as the variables representing the viscometric indices of power law lubricant are used by them for non-dimensionalisation. Obviating these controversies, Sinha et.al [64] derived a generalized Reynolds equation by assuming symmetry of flow of power law lubricant for lubrication of a cylinder on a plane. Prasad [96] considered rheological anomalies of the operating lubricant in the proximity of the bounding surface by considering the consistency of the lubricant film in the peripheral region different from that of

the film in the central region of the lubricant flow. The results were obtained by defining the load ratio as the ratio between loads with and without consideration of cavitation. Consideration of this load ratio parameter .. may lead to erraneous conclusions.

Motivated by the need to define the flow domain for the application of Reynolds equation for power law fluids and to study the effect of consistency variation across the film thickness in the proper perspective, in this work we employ the approach presented in the references [64, 96]. Consistency variation across the lubricant film is accounted through a general model for consistency variation which includes the variation of consistency due to thermal effects under the assumption of thermal equilibrium between the lubricant film and bounding bearing surfaces. This model [92-94] replaces the viscosity-temperature relationship by the viscosity-film thickness relationship. Such a replacement is possible in tribological conditions as it has been experimentally verified that the highest temperatures occur in zones where the film thickness is least [92].

The model proposed in this work enables us to study the effects of consistency variation across as well as along the film thickness. It has been employed to study problems of combined rolling and normal motion and elastohydrodynamic lubrication of rollers. The model finds its application for smooth as well as rough bearing surfaces in the subsequent Chapters. Though the Chapters progress with a unified theme, all of them are presented almost as independent entities except for direct reference to equations derived in the earlier Chapters.

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CHAPTER 2

GENERALIZED REYNOLDS EQUATION FOR POWER LAW LUBRICANTS WITH CONSISTENCY VARIATION: APPLICATION TO ROLLER BEARINGS UNDER LIGHTLY AND HEAVILY LOADED CONDITIONS

2.1 INTRODUCTION

Presence of additives in lubricants and consideration of high temperatures and pressures during bearing running-in give rise to variation of viscosity in the lubricating film across as well as along the film thickness [1-3]. Keeping this in view, several attempts have been made in the past two decades to generalize the Reynolds equation considering viscosity variation. Dowson [4] generalized this equation by considering the variation of fluid characteristics (viscosity and density) across as well as along the film thickness. Ezzat and Rhode [5] accounted viscosity variation through a viscosity profile considering thermal effects. There are experimental evidences to support the need to consider viscosity variation in the fluid film under certain conditions. For example, Askwith et.al [6] reported that the presence of additives in the base oil led to a formation of rigid plastic layer in the proximity of the bounding surface in elastohydrodynamic lubrication. The formation of high viscous layer very near to bearing surfaces has also been reported by Cameron and Gohar [1]. Dyson pointed out that the thickness of the enhanced viscous layer might be of the order of 10^{-8}-10^{-5} cm. and consideration of zones of enhanced

viscosity would have important effects in the lubrication of practical machinery [7].

It is noted that the study of viscosity variation is generally carried out for Newtonian lubricants ; scant attention has been given to this study for the case of non-Newtonian lubricants. Prasad [8] has employed power law model for the lubricant with additives and considered consistency variation across the film thickness by assuming that the consistency of the fluid film in the peripheral layer adjacent to the bearing surface is different from that of the central layer. Little attention has been given to study the effects of consistency variation which might arise due to adsorption of additives and variation of temperature. Although a bearing performance is adequately predictable isothermally in many cases, the experiments showed consistent descrepancies, however small, when correlated with the existing theories. These theories should be modified to account for thermal effects and flow conditions [9]. King et. al pointed out that a uniform temperature across the film appeared to be reasonable when small thicknesses and high velocities were involved. Most of the heat generated would be carried away by the lubricant while thermal conduction across the boundaries would play a minor role [10]. Keeping this in view, in this work, thermal equilibrium is assumed. Thermal effects, then, are accounted by assuming a relationship between consistency and film thickness akin to one proposed by Tipei [11]. Such

a relationship enables us to account for consistency variation along the film thickness in the undergoing analysis.

Thus, consideration of additives and thermal effects leads to consistency variation across as well as along the film thickness. Keeping this in view, in this chapter, a generalized form of Reynolds equation is derived. Subsequently, the effects of consistency variation on the lubrication characteristics of roller bearings are studied. These effects are studied in the case of rigid rollers under lightly loaded conditions in Sec 2.3; in Sec 2.4, the analysis has been extended to the regime of elastohydrodynamic lubrication to study these effects on the minimum film thickness.

2.2 GENERALIZED FORM OF REYNOLDS EQUATION

Consider the symmetrical flow of a power law lubricant between two identical rollers, each having a rolling velocity U (Fig. 2.1). The thickness, h, of the fluid film in the lubricated contact is small compared to the radius r of the rollers. With the usual assumptions of lubrication theory, the basic eqns. governing the flow of a power law lubricant (in one dimensional form) are given by [12]

$$\frac{dp}{dx} = \frac{\partial}{\partial y} \left\{ \left(m \left| \frac{\partial u}{\partial y} \right|^{n-1} \right) \frac{\partial u}{\partial y} \right\} \qquad (2.1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.2)$$

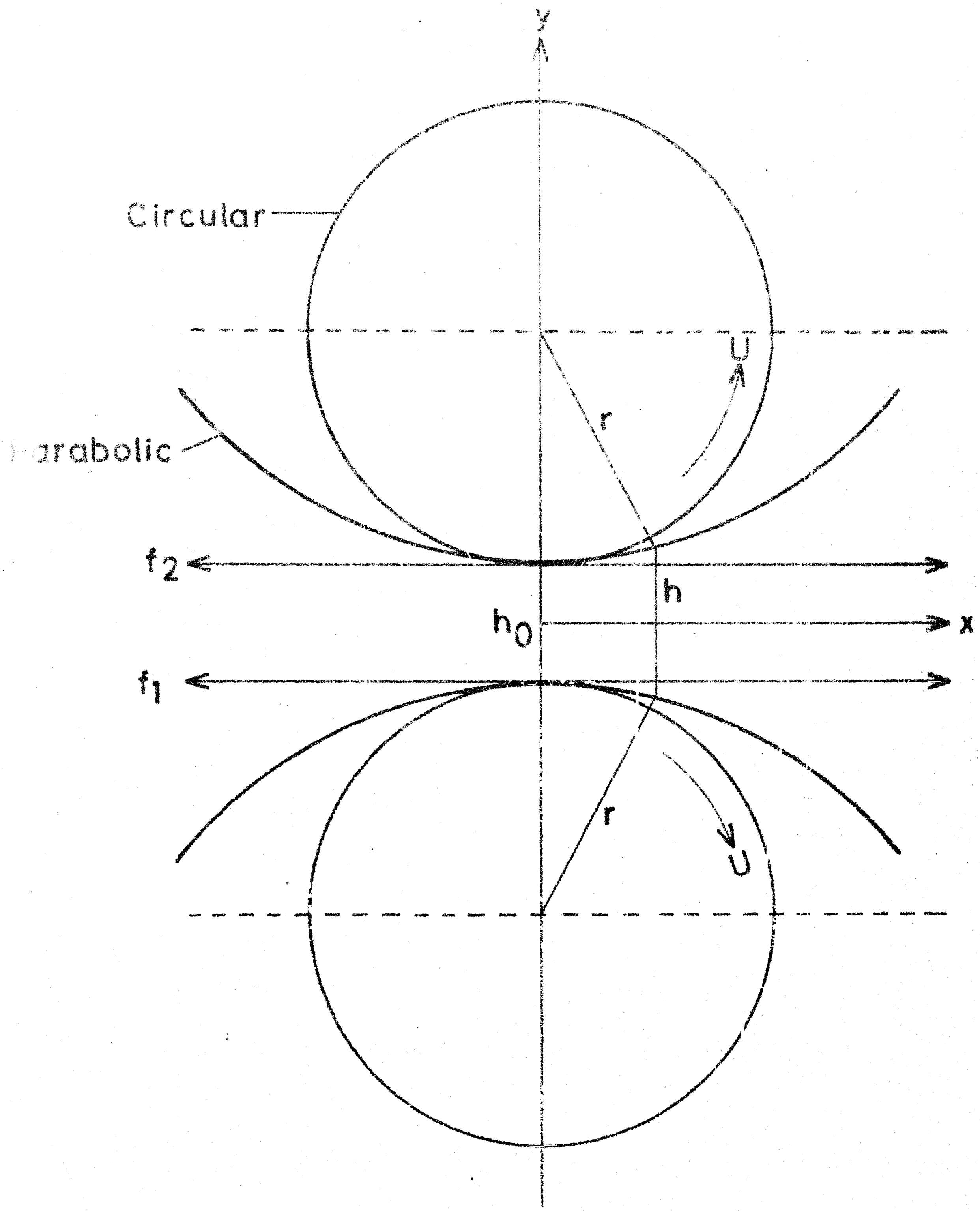

Fig. 2.1 Roller bearing configuration.

where u and v are velocities along the x and y directions, $p = p(x)$ is hydrodynamic pressure and $m = m(x,y)$ and n are viscometric parameters called consistency and flow behaviour indices, respectively. The lubricant behaviour is called pseudoplastic for $n < 1$, dilatant for $n > 1$ and Newtonian for $n = 1$.

As the system is symmetrical about x axis, it is sufficient to consider the fluid region $y \geq 0$ to determine velocities. Taking note of the symmetry, the following boundary conditions may be prescribed for u :

$$\begin{aligned} \frac{\partial u}{\partial y} &= 0 \quad \text{at } y = 0 \\ u &= U \quad \text{at } y = h/2 \end{aligned} \tag{2.3}$$

To determine the velocity u from eqn. (2.1), appropriate sign is to be attached to the velocity gradient $\frac{\partial u}{\partial y}$. To facilitate the determination of its sign, the velocity and pressure profiles are shown qualitatively in Fig. 2.2. It may be noted that the pressure becomes ambient at a point sufficiently away from the contact zone and reaches its maximum at a point $x = -x^*$. In other words,

$$\frac{dp}{dx} = 0 \quad \text{at } x = -x^* \tag{2.4}$$

Thus, in the region $-\infty \leq x \leq -x^*$, $\frac{\partial u}{\partial y} \geq 0$ and $\frac{dp}{dx} \geq 0$ and in the region $-x^* \leq x \leq x_c$, $\frac{\partial u}{\partial y} \leq 0$ and $\frac{dp}{dx} \leq 0$, where x_c is the point at which the film starts cavitating. Thus,

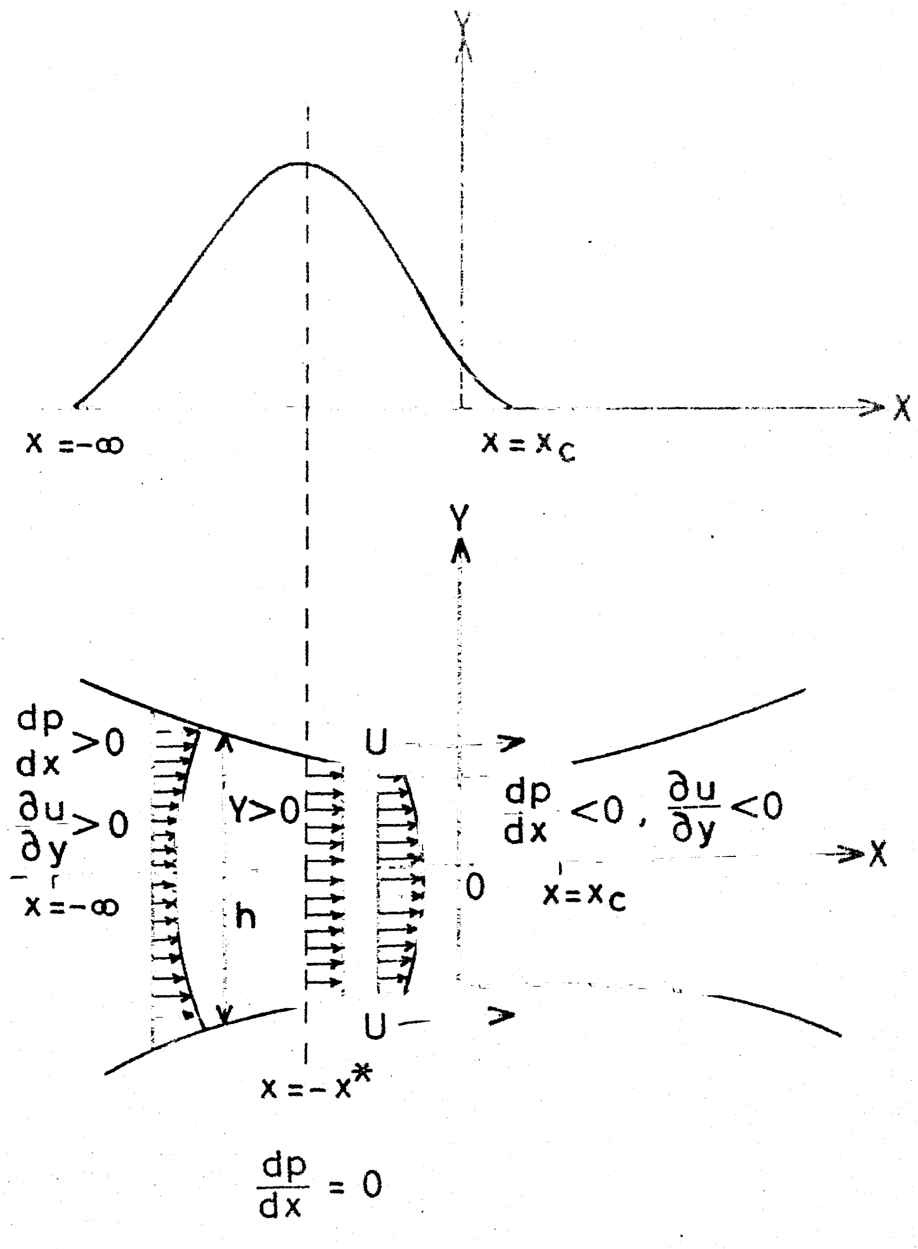

Fig. 2. 2 Lubrication of two rollers with power law lubricant.

the velocity distribution for the region $y \geq 0$ can be obtained as

$$u = U - \left(\frac{dp_1}{dx}\right)^{1/n} \int_y^{h/2} \left(\frac{y}{m}\right)^{1/n} dy \qquad -\infty \leq x \leq -x^* \qquad (2.5)$$

and

$$u = U + \left(-\frac{dp_2}{dx}\right)^{1/n} \int_y^{h/2} \left(\frac{y}{m}\right)^{1/n} dy \qquad -x^* \leq x \leq x_c \qquad (2.6)$$

where the pressure p is denoted by p_1 and p_2 in the regions defined in eqns. (2.5) and (2.6) respectively.

Now, integrating the equation of continuity (2.2) with the conditions

$$\begin{aligned} v &= 0 \quad \text{at } y = 0 \\ v &= V_{h/2} \text{ at } y = h/2 \end{aligned} \qquad (2.7)$$

we have,

$$-V_{h/2} = \frac{\partial}{\partial x} \int_0^{h/2} u\, dy - \frac{U}{2}\frac{dh}{dx} \qquad (2.8)$$

Note that $V_{h/2}$ is the resultant of the normal velocity V of the rollers and the normal velocity due to wedge action in the film segment, i.e.,

$$V_{h/2} = V + \frac{U}{2}\frac{dh}{dx} \qquad (2.9)$$

Positive values of V denote the normal separation of rollers while the negative values signify the normal approach implying squeezing action.

Substituting the expressions for u from eqns. (2.5) and (2.6) in eqn. (2.8), we obtain the basic eqns. determining pressure :

$$\frac{d}{dx}\{(f)(\frac{dp_1}{dx})^{1/n}\} = V + \frac{U}{2}\frac{dh}{dx} \qquad -\infty \leq x \leq -x^* \qquad (2.10)$$

$$\frac{d}{dx}\{(f)(-\frac{dp_2}{dx})^{1/n}\} = -(V + \frac{U}{2}\frac{dh}{dx}) \qquad -x^* \leq x \leq x_c \qquad (2.11)$$

where

$$(f) = \int_0^{h/2} y(\frac{y}{m})^{1/n}\, dy \qquad (2.12)$$

In order to evaluate the integral on the RHS of eqn. (2.12) we have to determine the function $m = m(x,y)$.

It has been mentioned earlier that the consistency of the lubricant layer adjacent to the bearing surface may be different from that of the central layer and it may vary with temperature as well. The viscosity of all liquids, especially hydrocarbon lubricating oils decreases rapidly with increase in temperature. This variation in viscosity with temperature is of practical importance in the lubrication of many mechanical devices such as gears, cams etc. where the lubricants are to function over a wide range of temperatures[13].

There is no fundamental mathematical relationship to predict accurately the variation in viscosity with temperature. The available viscosity-temperature relations are purely

empirical and the actual calculations require experimental data [14]. In this study, it is assumed that thermal equilibrium exists and the consistency varies according to a given law. To apply this law to real lubrication situations, the temperature at each point should be known. This requires a complete thermal calculation. However, a viscosity-temperature relationship can be replaced by a viscosity-film thickness relation. Such a replacement is possible in tribological conditions as it has been experimentally verified that the highest temperatures occur in zones where the film thickness is least [11]. In view of the viscosity-film thickness relation suggested in reference [11] , we assume that the consistency m of the power law lubricant varies according to the following law :

$$m = \bar{m}\left(\frac{h}{h_1}\right)^q \qquad (2.13)$$

where $\bar{m}$ is the consistency at the inlet film thickness h_1 where q, the thermal factor is measured.

q lies usually between 0 and 1. The value of q depends on the lubricant, the velocity of the bodies in contact, the cooling conditions and the flow of lubricant between bearing surfaces [11]. Thus, q can be determined by completing the thermal calculations. However, in the entire study of this work, various values are prescribed for q while examining the effects of consistency variation without going into details of thermal calculations.

It will be recalled that the rheological behaviour of the lubricant in the proximity of the solid surface results in an enhanced viscosity in the peripheral region. Particularly this phenomenon is evident when the additives in the lubricant are surface active and when the operating conditions of the lubricated contacts are in the elastohydrodynamic regime [7]. This results in consistency variation across the film thickness. Keeping this in view, Prasad [8] considered the consistency variation by assuming that the consistency varies as a step function across the film thickness. Taking these two aspects, viz. consistency variation along as well as across the film thickness into consideration, we define, now, consistency as,

$$\begin{aligned} m &= m_1 \left(\frac{h}{h_1}\right)^q \qquad && 0 \leq y < \frac{h}{2} - a \\ &= K m_1 \left(\frac{h}{h_1}\right)^q && \frac{h}{2} - a < y \leq \frac{h}{2} \end{aligned} \tag{2.14}$$

where K is the consistency ratio and a is the thickness of the peripheral layer.

Now, using eqn. (2.14) in eqn. (2.12) we obtain,

$$(f) = \frac{n}{2n+1} \left(\frac{1}{m_1}\right)^{1/n} \left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} \, h^{(2n+1-q)/n} (f_o) \tag{2.15}$$

where

$$(f_o) = 1-(1-K^{-1/n}) \{ 1-(1-2a/h)^{(2n+1)/n} \} \tag{2.16}$$

The value of (f) when substituted in eqns. (2.10) and (2.11) yields the generalized Reynolds eqn. which accounts for consistency variation :

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(f_o)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n}\right] = \frac{U}{2}\frac{dh}{dx} + V$$

$$-\infty \leq x \leq -x^*, y > 0 \qquad (2.17)$$

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(f_o)\left(-\frac{1}{m_1}\frac{dp_2}{dx}\right)^{1/n}\right]$$

$$= -\left(\frac{U}{2}\frac{dh}{dx} + V\right) \quad -x^* \leq x \leq x_c,\ y > 0 \qquad (2.18)$$

2.3 LUBRICATION OF LIGHTLY LOADED ROLLERS UNDER ROLLING

In this section, we study the effects of consistency variation on the bearing characteristics in the lubrication of two lightly loaded identical rollers under pure rolling conditions, with a power law lubricant, considering cavitation. The configuration of the system is as shown in Fig. 2.1. The Reynolds equation in this case, is written from eqns. (2.17) and (2.18) (putting V =0) :

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(f_o)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n}\right] = \frac{U}{2}\frac{dh}{dx}$$

$$-\infty \leq x \leq -x^* \qquad (2.19)$$

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(f_o)\left(-\frac{1}{m_1}\frac{dp_2}{dx}\right)^{1/n}\right] = -\frac{U}{2}\frac{dh}{dx}$$

$$-x^* \leq x \leq x_c \qquad (2.20)$$

To solve for pressure from eqns. (2.19) and (2.20), we use the following boundary conditions :

$$p_1 = 0 \quad \text{at } x = -\infty \tag{2.21}$$

$$\frac{dp_1}{dx} = \frac{dp_2}{dx} = 0 \text{ at } x = -x^* \tag{2.22}$$

$$p_1 = p_2 \qquad \text{at } x = -x^* \tag{2.23}$$

$$p_2 = \frac{dp_2}{dx} = 0 \text{ at } x = x_c. \tag{2.24}$$

Condition (2.21) indicates that the pressure is ambient at a distance sufficiently away from the contact zone in the inlet region where pressure build-up starts. This is dependent on the oil quantity supplied per unit time. When the oil flow is above a required level, the film starts sufficiently away from the point of contact. This point is virtually at infinity [15]. Eqns. (2.22) and (2.23) prescribe the matching conditions for pressure gradients and pressures,respectively, at $x = -x^*$, the point of maximum pressure .Condition (2.24) locates the point at which the film starts cavitating. In the general lubrication practice, it is assumed that the pressure is positive in the lubricated region and terminates at a point with zero pressure gradient where the film forms a discontinuous mixture of air, vapour and lubricant (in the cavitated region) [14]. At the point of cavitation, the continuity condition of zero pressure gradient is satisfied as has been shown by Floberg [15].

(2.24) represents usual cavitation boundary conditions applied extensively in bearing analysis.

Now, integrating eqns. (2.19) and (2.20) with the help of condition (2.22), we obtain the integrated form of the Reynolds equation as follows :

$$\frac{dp_1}{dx} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \frac{(h-h^*)^n}{(f_o)^n h^{2n+1-q}} \quad -\infty \leq x \leq -x^*, y > 0 \tag{2.25}$$

$$\frac{dp_2}{dx} = -\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \frac{(h^*-h)^n}{(f_o)^n h^{2n+1-q}} \quad -x^* \leq x \leq x_c, y > 0 \tag{2.26}$$

where the film thickness h is determined by approximating the rollers by a set of parabolic cylinders at the point of minimum film thickness and is given by

$$h = h_o + x^2/(2R) \tag{2.27}$$

where $R = r/2$ is the radius of the equivalent roller. Using eqns. (2.24) and (2.27) in eqn. (2.26), it can be seen that,

$$x_c = x^* \tag{2.28}$$

Thus, the points of maximum pressure and cavitation are symmetrical about the point of minimum film thickness.

Integrating eqns. (2.25) and (2.26) using conditions (2.21), (2.23) and (2.24), we obtain the expressions determining pressure :

$$p_1(x) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{x} \frac{(h-h^*)^n}{(f_o)^n h^{2n+1-q}} dx \qquad -\infty \leq x \leq -x^*,$$

$$y > 0 \qquad (2.29)$$

$$p_2(x) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-x^*} \frac{(h-h^*)^n}{(f_o)^n h^{2n+1-q}} dx - \int_{-x^*}^{x} \frac{(h^*-h)^n}{(f_o)^n h^{2n+1-q}} dx \right]$$

$$-x^* \leq x \leq x^*, \quad y > 0 \qquad (2.30)$$

In order to calculate the point of cavitation x^*, we use condition (2.24) in eqn. (2.30) which yields,

$$\int_{-\infty}^{-x^*} I_1(n,K,q)dx = \int_{-x^*}^{x^*} I_2(n,K,q)dx \qquad (2.31)$$

where

$$I_1(n,K,q) = \frac{(H-H^*)^n}{(F_o)^n H^{2n+1-q}}, \quad I_2(n,K,q) = \frac{(H^*-H)^n}{(F_o)^n H^{2n+1-q}}$$

$$(2.32)$$

and $(F_o) = 1-(1-K^{-1/n}) \{1-(1-2\bar{a}/H)^{(2n+1)/n}\}$ (2.33)

$$X = x/\sqrt{2Rh_o}, \quad X^* = x^*/\sqrt{2Rh_o}, \quad H = h/h_o = 1+X^2, \quad H^* = h^*/h_o = 1+X^{*2},$$

$$\bar{a} = a/h_o \qquad (2.34)$$

The solution of eqn. (2.31) gives the point of cavitation.

The load component $W_x(n,K,q)$ per unit length (in the x direction) on the cylinder is given by,

$$W_x(n,K,q) = \int_{-\infty}^{x^*} \frac{x^2}{R} \frac{dp}{dx} dx \qquad (2.35)$$

Using eqns. (2.25) and (2.26) in eqn. (2.35) we obtain,

$$W_x(n,K,q)=\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 \frac{U^n}{R} \left(\frac{1}{h_1}\right)^q \Big[\int_{-\infty}^{-x^*} x^2 \frac{(h-h^*)^n}{(f_o)^n h^{2n+1-q}} dx$$

$$- \int_{-x^*}^{x^*} x^2 \frac{(h^*-h)^n}{(f_o)^n h^{2n+1-q}} dx\Big] \quad (2.36)$$

The load component $W_y(n,K,q)$ in the y direction is given by,

$$W_y(n,K,q) = \int_{-\infty}^{x^*} p\, dx = - \int_{-\infty}^{x^*} x \frac{dp}{dx} dx \quad (2.37)$$

which on using eqns. (2.25) and (2.26) yields,

$$W_y(n,K,q) = -\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-x^*} x \frac{(h-h^*)^n}{(f_o)^n h^{2n+1-q}} dx$$

(2.38)

To study the effects of consistency variation across the film thickness on load, we define the quantities $\bar{W}_{Kx}$ and $\bar{W}_{KY}$ as follows :

$$\bar{W}_{KX} = \frac{W_x(n,K,q)}{W_x(n,1,q)} = \frac{\int_{-\infty}^{-X^*} X^2 I_1(n,K,q)\,dX - \int_{-X^*}^{X^*} X^2 I_2(n,K,q)\,dX}{\int_{-\infty}^{-X^*_{K1}} X^2 I_1(n,K,q)\,dX - \int_{-X^*_{K1}}^{X^*_{K1}} X^2 I_2(n,K,q)\,dX} \quad (2.39)$$

and

$$\bar{W}_{KY} = \frac{W_y(n,K,q)}{W_y(n,1,q)} = \frac{\int_{-\infty}^{-X^*} X I_1(n,K,q)\,dX}{\int_{-\infty}^{-X^*_{K1}} X I_1(n,1,q)\,dX} \quad (2.40)$$

where the expressions $I_1(n,K,q)$ and $I_2(n,K,q)$ are defined in eqn. (2.32).

The case $K = 1$ indicates no variation of consistency across the fluid film ; that is, the absence of peripheral layer in the lubricant film. It gives the consistency variation along the film thickness only. The points $-X^*_{k1}$ and X^*_{k1} denote the points of maximum pressure and cavitation, respectively, for the case $k = 1$.

The effect of consistency variation along the film thickness on load can be studied by defining the quantities $\bar{W}_{qX}$ and $\bar{W}_{qY}$ as follows :

$$\bar{W}_{qX} = \frac{W_x(n,K,q)}{W_x(n,K,O)}$$

$$= \frac{\int_{-\infty}^{-X^*} X^2 I_1(n,K,q)/H_1^q \, dX - \int_{-X}^{X^*} X^2 I_2(n,K,q)/H_1^q \, dX}{\int_{-\infty}^{-X^*_{q_o}} X^2 I_1(n,K,O) dX - \int_{-X^*_{qo}}^{X^*_{q_o}} X^2 I_2(n,K,O) dx} \tag{2.41}$$

$$\bar{W}_{qY} = \frac{W_y(n,K,q)}{W_y(n,K,O)} = \frac{\int_{-\infty}^{-X^*} X\, I_1(n,K,q)/H_1^q \, dX}{\int_{-\infty}^{-X^*_{q_o}} XI_1(n,K,O) dx} \tag{2.42}$$

The case $q = 0$ corresponds to the isothermal considerations and $H_1=h_1$.

Thus, it gives the consistency variation across the film thickness only.

The points $-X^*_{qo}$ and X^*_{qo} represent the points of maximum pressure and cavitation, respectively, for the case $q = 0$. The expression for frictional force on the surface $y = h/2$ is given by,

$$f_2 = \int_{-\infty}^{x^*} \tau\Big|_{y=h/2} dx \tag{2.43}$$

where

$$\tau = m\left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y}. \tag{2.44}$$

τ, being the shear stress.

Using the expressions for u from eqns. (2.5), (2.6) and (2.44) in eqn. (2.43), we obtain

$$f_2 = -\left[\int_{-\infty}^{-x^*} \frac{h}{2}\frac{dp_1}{dx}dx + \int_{-x^*}^{x^*} \frac{h}{2}\frac{dp_2}{dx}dx\right] \tag{2.45}$$

With the help of eqn. (2.35) it can be seen that,

$$f_2(n,K,q) = -W_x(n,K,q)/4 \tag{2.46}$$

Similarly, the frictional force $f_1(n,K,q)$ on the surface $y = h/2$ is given by,

$$f_1(n,K,q) = -W_x(n,K,q)/4 \tag{2.47}$$

Thus we have,

$$f_1(n,K,q) + f_2(n,K,q) + W_x(n,K,q)/2 = 0 \tag{2.48}$$

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This is the condition for overall equilibrium that must be applied to the lubrication of roller bearings in the absence of inertia and body forces. Substituting in eqn. (2.48) $k = 1$ and $q = 0$ we obtain the result of reference [16] in the case of Newtonian lubricants.

2.4 ELASTOHYDRODYNAMIC (EHD) LUBRICATION

In this section, we study the effects of consistency variation on the minimum film thickness in the ehd lubrication of heavily loaded rollers. The analysis is restricted to the inlet region. For this purpose, we use Grubin's theory [17] and Crook's approximation [18] in the analysis. In Grubins theory two simplifying assumptions are made [16]: (1) The deformed shape of the bearing surface is the same as that in the dry contact. (2) A high pressure is developed in the entry region to the main loading region, called Hertzian zone. The physical situation is as shown in Fig. 2.3 where 2d is the width of the Hertzian region and the pressure is maximum just outside this zone, i.e., at $x = -d$.

The eqn. to determine pressure p_E in the inlet zone of ehd regime is written from eqn. (2.19) with p_1 replaced by p_E as,

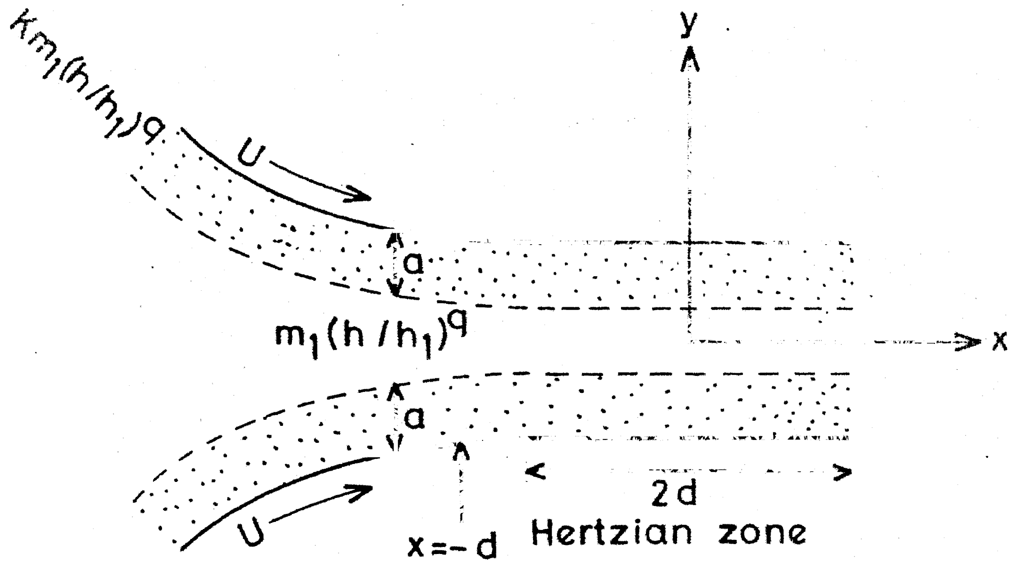

Fig. 2.3 Elastohydrodynamic lubrication between two rollers considering consistency variation, $h_m \geq 2a$

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}\, (f_o)\left(\frac{1}{m_1}\frac{dp_E}{dx}\right)^{1/n}\right] = \frac{U}{2}\frac{dh}{dx}$$

$$-\infty \leq x \leq -d \qquad (2.49)$$

where [19]

$$d = 2[2W_E R_E/(\pi E)]^{1/2},\ 1/E = (1-\nu^2)/E_1,\ R_E = r_E/2 \qquad (2.50)$$

ν is the Poisson's ratio, E and E_1 are the Young's moduli., R_E is the radius of the equivalent roller, r_E is the radius of the lubricated roller and W_E is the load in the ehd case. To determine pressure in the inlet region from eqn. (2.49) we use the following boundary conditions :

$$p_E = 0 \quad \text{at } x = -\infty \qquad (2.51)$$

$$p_E = \infty,\quad \frac{dp_E}{dx} = 0 \text{ at } x = -d \qquad (2.52)$$

Integrating eqn. (2.49) with the second condition of (2.52), we obtain

$$\frac{dp_E}{dx} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1}\, m_1 U^n \left(\frac{1}{h_1}\right)^q \frac{(h-h_m)^n}{(f_o)^n\, h^{2n+1-q}} \qquad (2.53)$$

where $h = h_m$ is the minimum film thickness at $x = -d$ which is constant along the Hertzian zone. To write an expression for the film thickness in the heavily loaded lubricated contact, we first note that the deformed shape of the rollers outside the band of Hertzian contact is given by [20]

$$h_2=(d^2/(2R))\left[\,|x/d|(x^2/d^2-1)^{1/2}-\ln\left\{|x/d|+(x^2/d^2-1)^{1/2}\right\}\right] \qquad (2.54)$$

which may be approximated as [18]

$$h_3 = 2^{3/2}\ d^2(3R)^{-1}\ \varepsilon^{3/2} + \text{higher powers of } \varepsilon \qquad (2.55)$$

where ε is given by

$$|x| = d(1+\varepsilon) \qquad (2.56)$$

According to Grubin's theory, in the Hertzian region the bearing surfaces are separated by a constant film thickness. Hence, the film thickness in the ehd contact of rollers is given by the deformed shape of the rollers outside this region with the addition of constant separation in the Hertzian region. Thus, we have

$$h = 2^{3/2}\ d^2(3R)^{-1}\ \varepsilon^{3/2} + h_m \qquad (2.57)$$

In the ehd lubrication the consistency of the lubricant is assumed to vary exponentially with pressure [20,21]:

$$m_1 = m_o\ e^{\alpha p} \qquad (2.58)$$

where α is the pressure coefficient of consistency index m_1. This type of eqn. is beleived to give a fairly accurate viscosity-pressure relation in the case of high viscosity fluids under isothermal considerations [22]. The consideration of consistency variation with pressure under isothermal conditions in eqn. (2.58) is valid, for, in the definition of m in eqn. (2.14) m_1 may principally be a function of pressure [11].

Now, integrating eqn. (2.53) using eqns. (2.54)-(2.58) we get,

$$\frac{1}{\alpha} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_o U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-d} \frac{(h-h_m)^n}{(f_o)^n h^{2n+1-q}} dx \quad (2.59)$$

On simplifying, the formula for minimum film thickness in the case of ehd lubrication can be written as follows :

$$\left(\frac{h_m}{R}\right)^{(3n+1)/3} = C_1 \alpha m_o \left(\frac{1}{H_1}\right)^q I_{E1}(n,K,q,0) \quad (2.60)$$

where

$$C_1 = (2n+1)^n 2^{3/2} 3^{-1/3} \pi^{1/6} \left(\frac{W}{ER}\right)^{-1/6} \left(\frac{2U}{nR}\right)^n, \quad H_1 = \frac{h_1}{h_m} \quad (2.61)$$

$$I_{E1}(n,k,q,\theta) = \int_{\theta}^{\pi/2} (\cos\theta)^{(6n-6q-1)/3} (\sin\theta)^{(6n+1)/3} / (F_o)^n \, d\theta \quad (2.62)$$

$$(F_o) = 1-(1-K^{-1/n})\{1-(1-\beta\cos^2\theta)^{(2n+1)/n}\} \quad (2.63)$$

$$h = h_m \sec^2\theta, \quad \beta = \frac{2a}{h_m}, \quad d^2(2\varepsilon)^{3/2}(3Rh_m)^{-1} = \tan^2\theta \quad (2.64)$$

Eqn. (2.60) gives an expression for the minimum film thickness in the case of $h_m \geq 2a$. The minimum thickness formula for the case of no consistency variation across the film thickness ($k = 1$) is written from eqn. (2.60) as,

$$\left(\frac{h_{m(K=1)}}{R}\right)^{(3n+1)/3} = C_1 \alpha m_o \left(\frac{1}{H_1}\right)^q I_{E2}(n,q,\pi/2) \quad (2.65)$$

where

$$I_{E2}(n,q,\theta) = \int_0^{\theta} (\cos\theta)^{(6n-6q-1)/3} (\sin\theta)^{(6n+1)/3} d\theta \quad (2.66)$$

Using eqns. (2.60) and (2.65) we write the ratio $\frac{h_m}{h_{m(k=1)}}$ in the case of $h_m \geq 2a$ as,

$$\frac{h_m}{h_m(K=1)} = [I_{E1}(n,K,q,0)/I_{E2}(n,q,\pi/2)]^{3/(3n+1)} \quad (2.67)$$

The expression for minimum thickness formula with no consistency variation along the film thickness is given by $h_m(q=0)$. Hence, we write the ratio $h_m/h_m(q=0)$ for $h_m \geq 2a$ as

$$h_m/h_m(q=0) = [(1/H_1)^q \, I_{E1}(n,K,q,0)/I_{E1}(n,K,0,0)]^{3/(3n+1)} \quad (2.68)$$

When $h_m \leq 2a$, the inlet zone is divided into two regions $-\infty \leq x \leq -d_1$, $h \geq 2a$ and $-d_1 \leq x \leq -d$, $h \leq 2a$ as shown in Fig.2.4 where $h = 2a$ at $x = -d_1$.

The governing pressure eqn. for the region $-\infty \leq x \leq -d_1$ is rewritten from eqn. (2. 49) by taking p_{1E} for p_E as

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} \, h^{(2n+1-q)/n}(f_o)\left(\frac{1}{m_1}\frac{dp_{1E}}{dx}\right)^{1/n}\right] = \frac{U}{2}\frac{dh}{dx} \quad (2.69)$$

In the region $-d_1 \leq x \leq -d$, the consistency of the fluid film varies as $Km_1(h/h_1)^q$ in the entire region. The eqn. for pressure p_{2E} in this region can be written as

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} \, h^{(2n+1-q)/n}\left(\frac{1}{Km_1}\frac{dp_{2E}}{dx}\right)^{1/n}\right] = \frac{U}{2}\frac{dh}{dx} \quad (2.70)$$

The boundary conditions for eqns. (2.69) and (2.70) can be written as

$$\frac{dp_{1E}}{dx} = \frac{dp_{2E}}{dx} \quad \text{at } x = -d_1 \text{ or } h = 2a \quad (2.71)$$

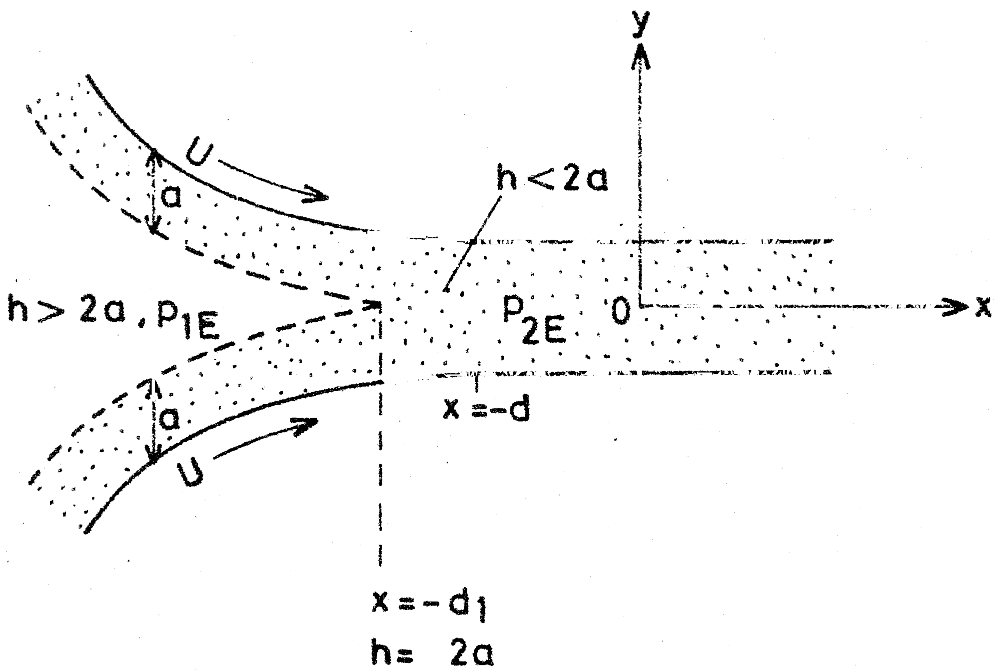

Fig.2.4 Elastohydrodynamic lubrication between two rollers, considering consistency variation, $h_m \leq 2a$.

$$p_{1E} = 0 \text{ at } x = -\infty \tag{2.72}$$

$$p_{1E} = p_{2E} = p_m \text{ at } x = -d_1 \tag{2.73}$$

$$\frac{dp_{2E}}{dx} = 0,\ p_{2E} = \infty \text{ at } x = -d \tag{2.74}$$

Integrating eqn. (2.69) we get

$$\frac{dp_{1E}}{dx} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1}\, m_1 U^n \left(\frac{h}{h_1}\right)^q \times$$

$$\times \left[\frac{h+h_c}{h^{(2n+1)/n} - \{(1-K^{-1/n})(h^{(2n+1)/n} - (h-2a)^{(2n+1)/n}\}}\right]^n \tag{2.75}$$

where h_c is the constant of integration to be determined.

Integrating eqn. (2.70) and using condition (2.74)

$$\frac{dp_{2E}}{dx} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1}\, k m_1 U^n \left(\frac{h}{h_1}\right)^q \left[\frac{h-h_m}{h^{(2n+1)/n}}\right]^n \tag{2.76}$$

where $h = h_m$ at $x = -d$.

Now, using condition (2.71) in eqns. (2.75) and (2.76) we get $h_c = -h_m$

Recollecting that $m_1 = m_o\, e^{\alpha p}$, we use $m_1 = m_o e^{\alpha p_{1E}}$ in the region $-\infty \leq x \leq -d_1$ and $m_1 = m_o e^{\alpha p_{2E}}$ in the region $-d_1 \leq x \leq d$ and integrating eqns. (2.75) and (2.76) using conditions (2.72)-(2.74) we get

$$1-e^{-\alpha p_m} = \alpha C_1 m_o (R/h_m)^{(3n+1)/3}\, I_{E1}(n,K,q,\theta)/H_1^q \tag{2.77}$$

$$e^{-\alpha p_m} = K\alpha C_1 m_o (R/h_m)^{(3n+1)/3}\, I_{E2}(n,q,\theta_1)/H_1^q \tag{2.78}$$

where $C_1, I_E(n,K,q,\theta_1)$ and $I_{E2}(n,q,\theta_1)$ are given by eqns. (2.61), (2.62) and (2.66) respectively, and

$$\theta_1 = \cos^{-1}(\beta^{-1/2}),\ \beta = 2a/h_m. \tag{2.79}$$

Adding eqns. (2.77) and (2.78) we have

$$\left(\frac{h_m}{R}\right)^{(3n+1)/3} = \alpha C_1 m_o [I_{E1}(n,K,q,\theta_1)+KI_{E2}(n,q,\theta_1)]/H_1^q \tag{2.80}$$

Eqn. (2.80) gives the mini-film thickness formula for $h_m \le 2a$. We define the ratios $h_m/h_m(K=1)$ and $h_m/h_m(q=0)$ in this case as well;

$$\frac{h_m}{h_m(K=1)} = \left[\frac{I_{E1}(n,K,q,\theta_1)+KI_{E2}(n,q,\theta_1)}{I_{E2}(n,q,\pi/2)}\right]^{3/(3n+1)} \tag{2.81}$$

$$\frac{h_m}{h_m(q=0)} = \left[\left(\frac{1}{H_1}\right)^q \frac{I_{E1}(n,K,q,\theta_1) + KI_{E2}(n,q,\theta_1)}{I_{E1}(n,K,0,\theta_1)+KI_{E2}(n,0,\theta_1)}\right]^{3/(3n+1)} \tag{2.82}$$

We see that putting $q = 0$ in eqns. (2.67) and (2.81), we obtain the minimum thickness formulae for $h_m \le 2a$ and $h_m \le 2a$ as shown in reference [8].

2.5 RESULTS AND DISCUSSION

Eqn. (2.31) is numerically solved to determine the point of cavitation, X^*, for rigid rollers, for various values of the flow behaviour index n, the consistency ratio K, the peripheral layer thickness $\bar{a}$ and the thermal factor q. The load ratio parameters $\bar{W}_{KX}$, $\bar{W}_{KY}$, $\bar{W}_{qX}$ and $\bar{W}_{qY}$ are, then, calculated by using eqns (2.39)-(2.42). The case $K > 1$ signifies an increase in the consistency of the lubricant in the peripheral region, increasing thereby the effective consistency in the lubrication process relative to the case with no peripheral layer. The case $K < 1$ indicates a decrease in the effective consistency. The latter situation may arise with certain additives when chemical reaction takes place near the bounding solid surface. This will cause the additive particles to disappear from the bounding surface or the lubricating film [23]. Davenport [24] pointed out that higher viscosity might be due to the presence of additives or surfactants in the carrier oil. This condition, in our study, pertains to the case of $K > 1$.

Increasing value of q signifies a decrease in the consistency. This may be a consequence of temperature rise as it is well known that temperature rise results in viscosity/consistency reduction. The case $q = o$ represents the isothermal consideration, and provides maximum pressure generation and load capacity.

Throughout this work, we represent thermal effects by q, and call it as thermal factor. Setting q = o in the present analysis, we obtain the analysis of Prasad [8]. However, his results cannot be compared to that obtained in this study because the load ratios $\bar{W}_{KX}$ and $\bar{W}_{KY}$ defined by him carry dual consideration of cavitation and no cavitation. These ratios lead to erraneous conclusions for, consideration and disregarding of cavitation will alter the load capacity significantly. In the present analysis, the ratios are calculated considering cavitation only. The value of n normally lies between 0.5 and 2.5 [25]. For the purpose of comparison of pseudoplastic and dilatant behaviours, the values of n = 0.5 and n = 1.5 have been taken as the representative values.

Fig.2.5 gives the picture of the location of cavitation point with respect to specified K for various values of n. As K increases, it can be observed that X^* moves towards the point of minimum film thickness; however, the shifting towards the converging gap is very small for increasing values of large K. Further, for any fixed K, the cavitation point for dilatants lies closer to the point of minimum film thickness as compared to pseudoplastics.

From Fig. 2.6, we observe that for K< 1, an increase in $\bar{a}$ results in a shift away of X^* from the point of minimum film thickness for all n ; for K > 1, the trend of shifting

is reversed.

The effect of q on the location of X^* is observed in Fig.2.7. An increase in the value of q shifts the point of cavitation towards the region of diverging film segment. As pointed out earlier, increasing q signifies reduction in the consistency and pushes the cavitation point towards the diverging gap. It is observed from the Figure that the effect of q on X^* is more pronounced in the case of dilatants compared to pseudoplastics for large values of q.

Figs. 2.8 and 2.9 depict the effect of K on load ratio parameters $\bar{W}_{KX}$ and $\bar{W}_{KY}$. From these Figures, we observe that the effect of K on load capacity for dilatants is much more pronounced compared to that for pseudoplastics. This effect gives greater load capacity for $K > 1$ and less load capacity for $K < 1$ for dilatants compared to pseudoplastics. The effect of increasing $\bar{a}$ in the peripheral layered film thickness ($K > 1$) provides greater load capacity compared to the case with no peripheral layer. A reversed trend of load capacity is observed in the case of $K < 1$ (Figs. 2.10, 2.11).

From Figs. 2.12 and 2.13, we observe that as q increases the load capacity decreases for all n. The decrease is more for dilatants compared to pseudoplastics for large values of q.

The effect of consistency ratio K and thermal factor q on the minimum film thickness in the ehd lubrication are studied in Figs. 2.14-2.17. To study the effect of K, eqns. (2.67) and (2.81) are numerically evaluated for $h_m/2a \geq 1$ and $h_m/2a \leq 1$ respectively for specified values of $h_m/2a$ for K = 1. From Fig. 2.14 we observe that the effect of K is to increase the minimum film thickness. For K > 1, the film thickness will be higher than that of the case with no peripheral layered ehd minimum film thickness and for K < 1 the film thickness is smaller.

The effect of K on minimum film thickness for various values of n is manifested in Fig.2.15. For K < 1, the increase in the minimum film thickness is more for pseudoplastics compared to that dilatants. This behaviour is in qualitative agreement with the findings of Prasad [8]. The trend is reversed in the case of K < 1 when $h_m(K=1)/2a$ is sufficiently large. It is to be noted that no definite pattern of the flow behaviour is observed when the peripheral layer is close to the half of minimum film thickness in the case of K < 1.

The effect of q on minimum film thickness is studied with eqns. (2.68) and (2.82). An increase in the value of q for K > 1 reduces the minimum film thickness (Fig.2.16).

However for $K < 1$, there is slight increase in the minimum film thickness(Fig 2.17). One may infer that for $K < 1$ an increase in the thermal factor has a beneficial effect on the ehd minimum film thickness. The calculations showed that in the case of ehd lubrication, a reasonal prediction of film thickness is not always possible with high values of q. Hence the formulae for minimum film thickness will be reliable for low values of q, particularly for isothermal case.

2.6 CONCLUSIONS

In this Chapter, a generalized one dimensional Reynolds equation is derived for power law lubricants with consistency varying across and along the film thickness. Consideration of consistency variation across the film thickness is due to the presence of peripheral layer and this variation along the film thickness is due to thermal effects.

The effect of the high consistency peripheral layer is to shift the cavitation point towards the point of minimum film thickness and to increase the load capacity with respect to flow behaviour index. The trend is reversed for the case with low consistency peripheral layer. The effect of thermal effects is to decrease the load capacity; the decrease is more for dilatants than for pseudoplastics.

For heavily loaded rollers, the effect of high consistency peripheral layer is to increase the minimum film thickness; the increase is more for pseudoplastics compared to dilatants. The trend is reversed for low peripheral layer film thickness, provided the film thickness is sufficiently high.

No set pattern of the flow behaviour is observed when the peripheral layer is close to half of ehd film. The effect of thermal factor is to decrease the minimum film thickness in the case of high consistency peripheral layer. The trend is reversed for low consistency peripheral layer.

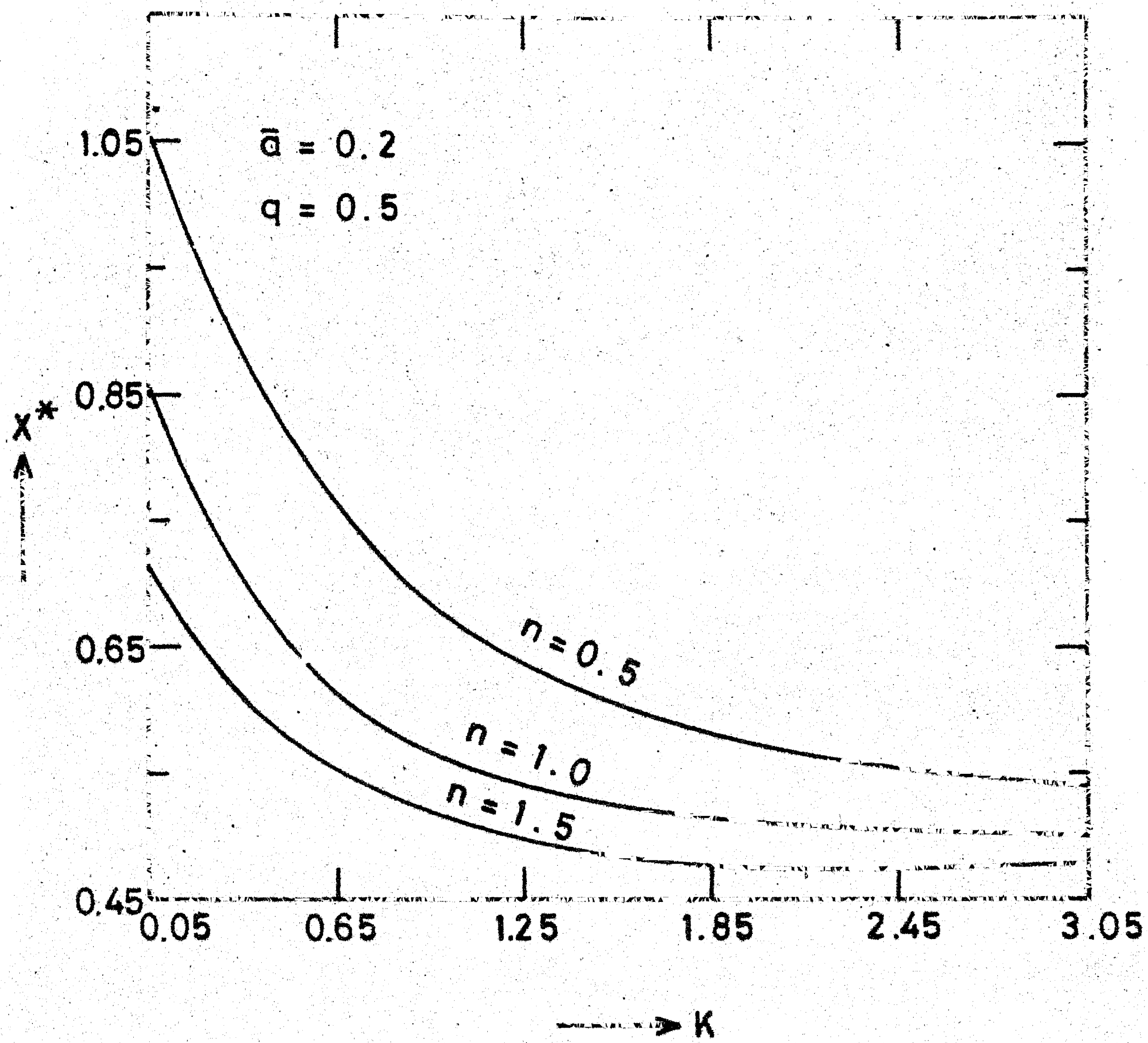

Fig. 2.5 Cavitation point X^* vs. consistency ratio K for various values of flow behaviour index n.

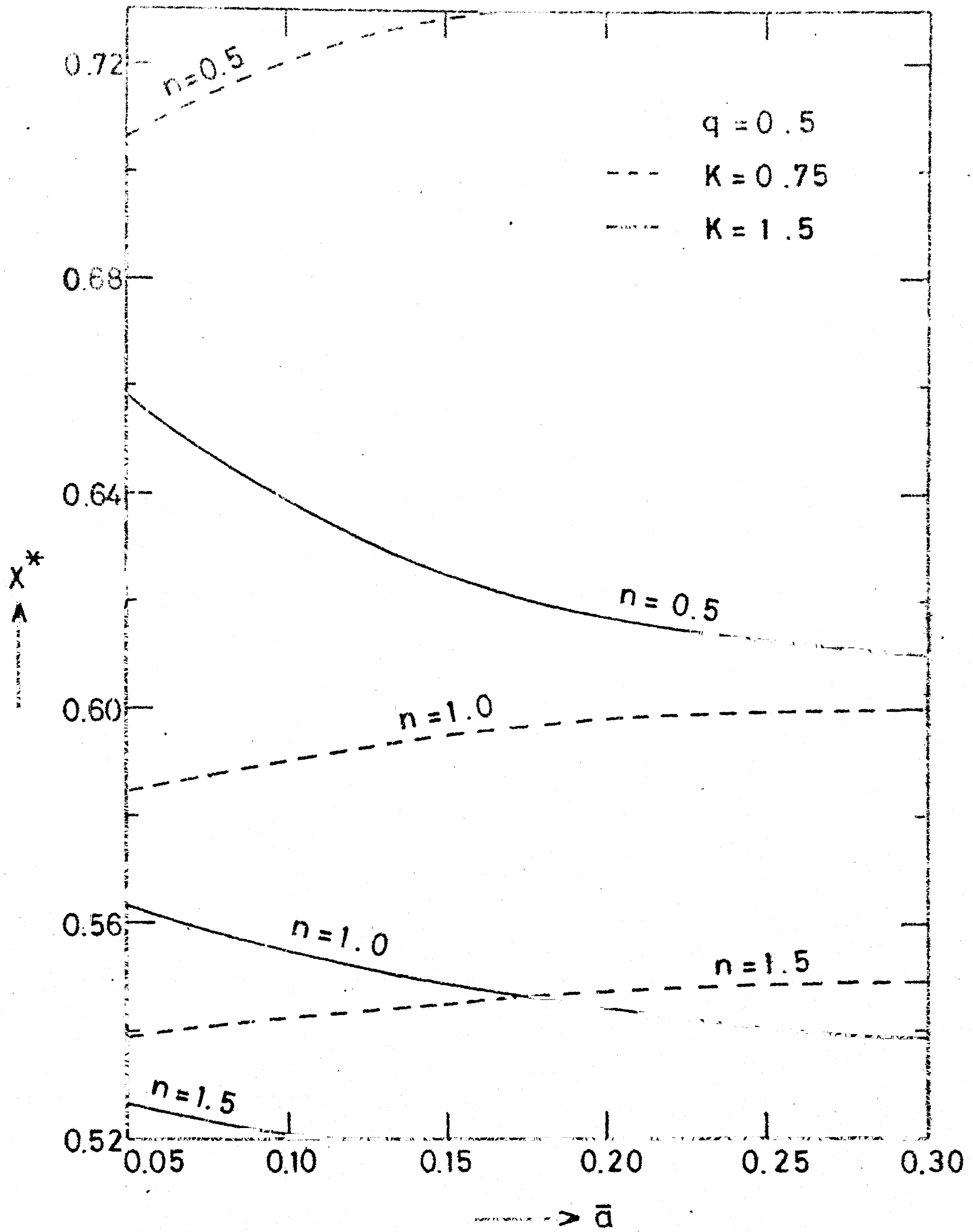

Fig. 2.6 Cavitation point X^* vs. peripheral layer thickness $\bar{a}$ for various values of flow behaviour index n.

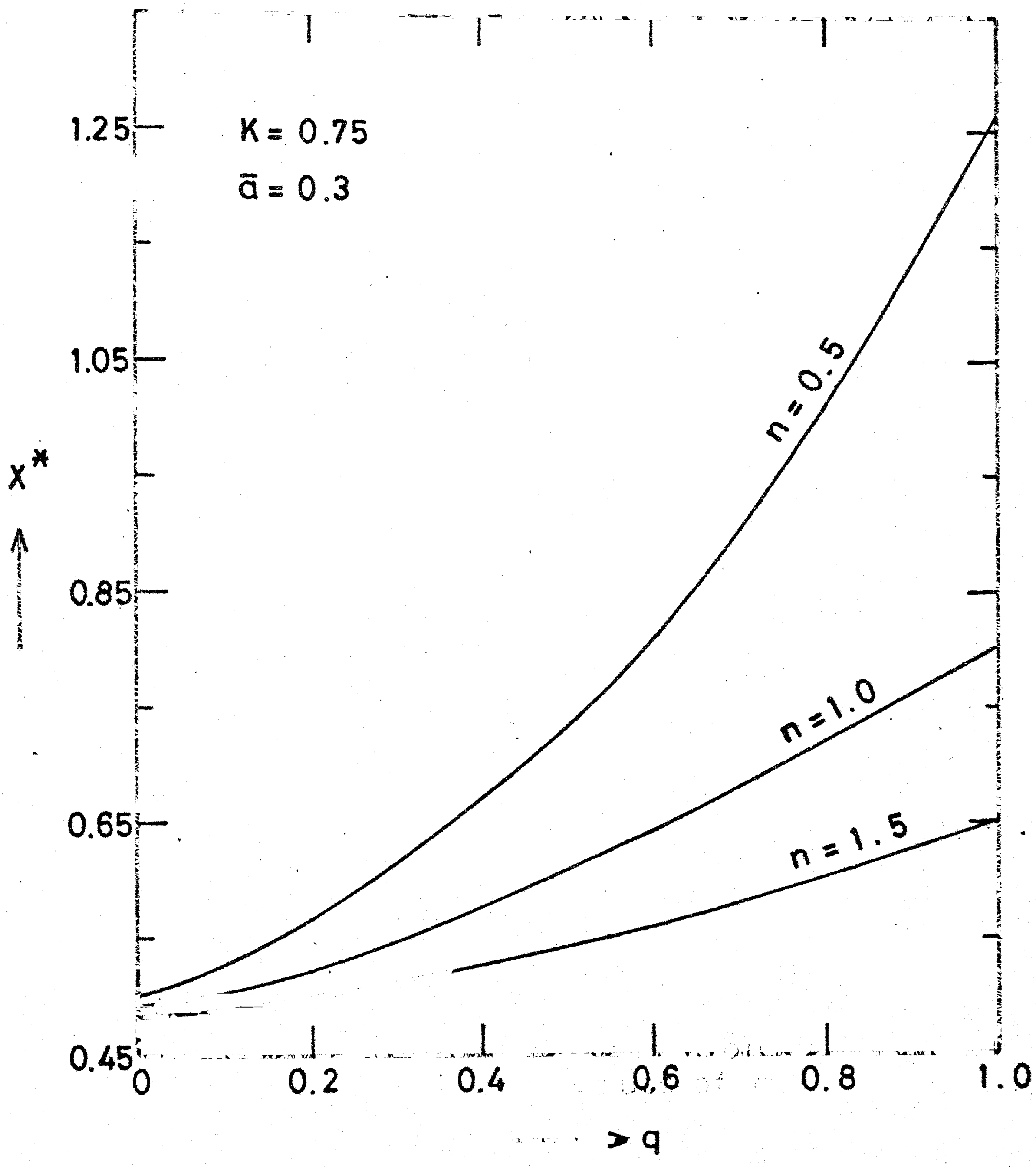

Fig. 2.7 Cavitation point X^* vs. thermal factor q for various values of flow behaviour index n.

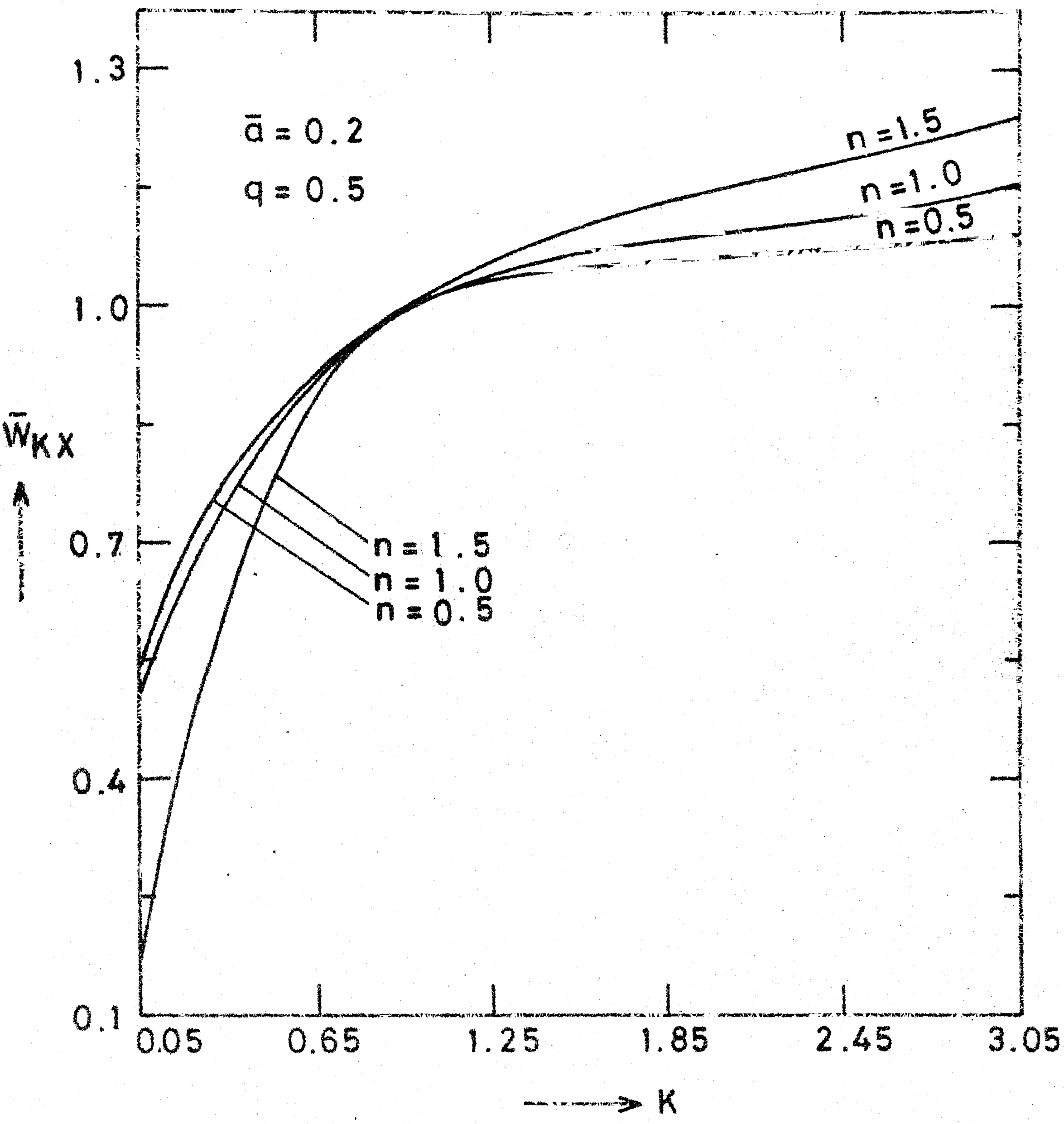

Fig. 2.8 Load capacity ratio $\bar{W}_{KX}$ vs. consistency ratio K for various values of flow behaviour index n.

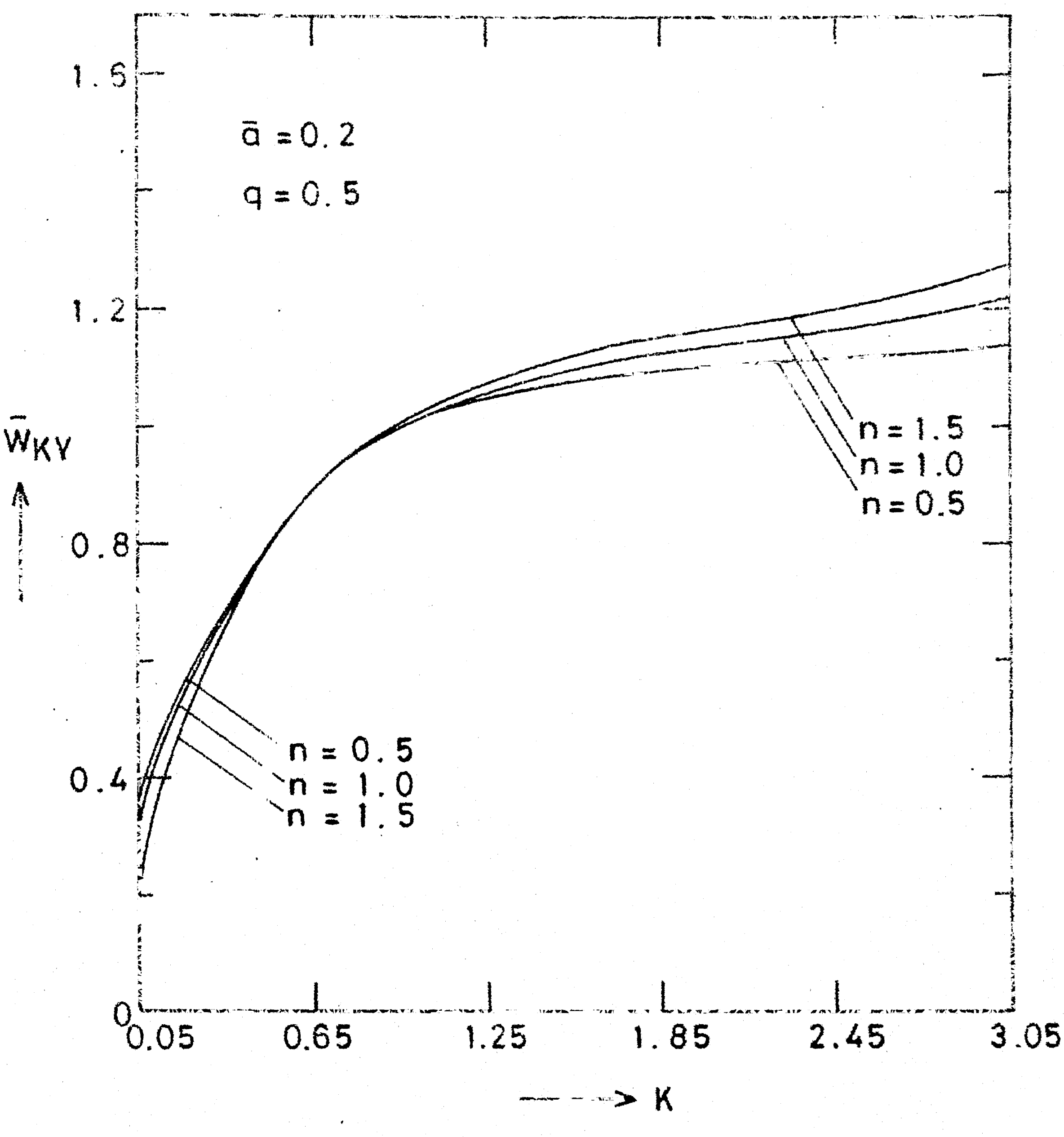

Fig. 2.9. Load capacity ratio $\bar{W}_{KY}$ vs. consistency ratio K for various values of flow behaviour index n.

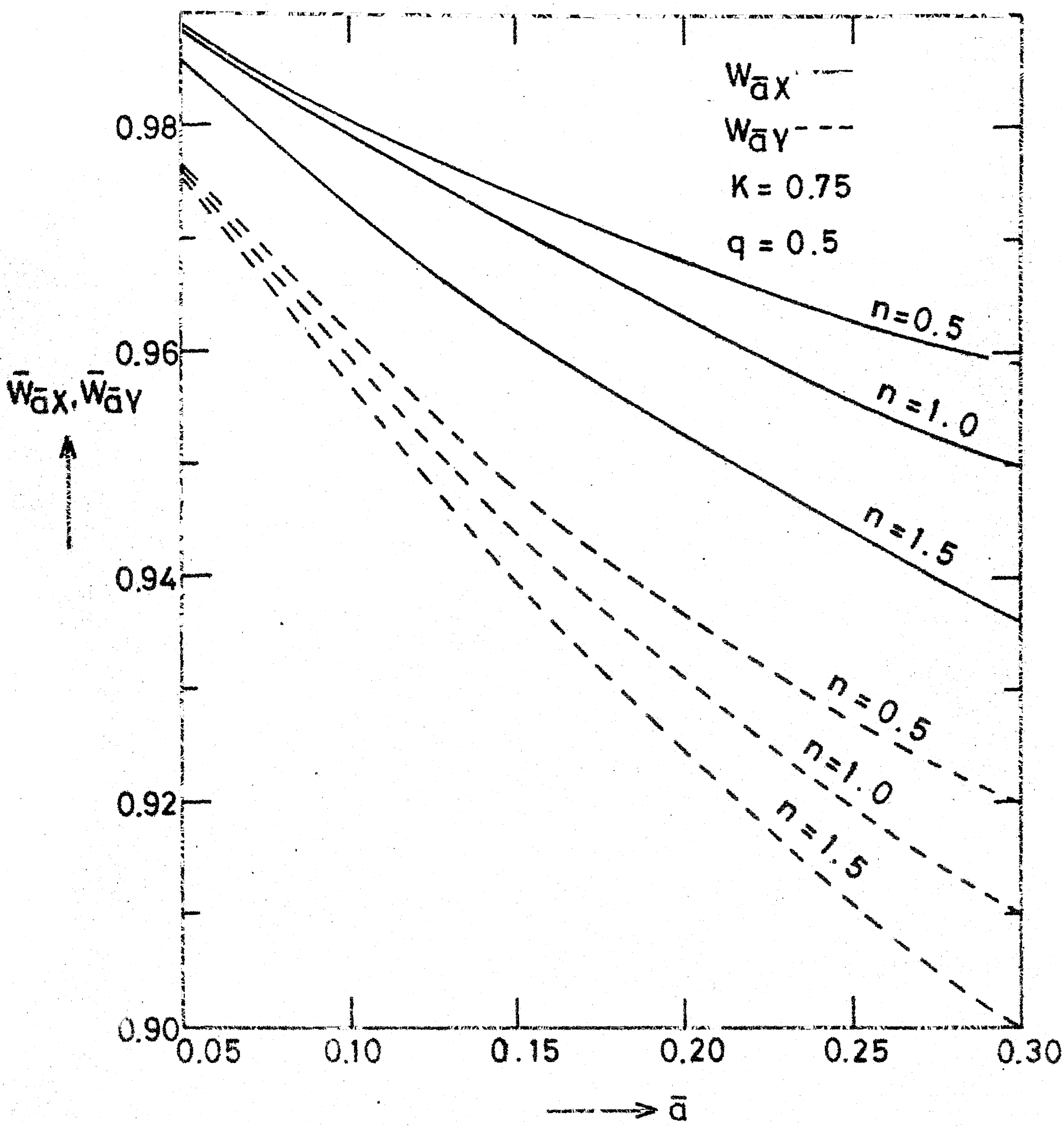

Fig. 2.10 Load capacity ratios $\bar{W}_{\bar{a}X}$ and $\bar{W}_{\bar{a}Y}$ peripheral layer thickness $\bar{a}$ for various values of flow behaviour index n, K > 1.

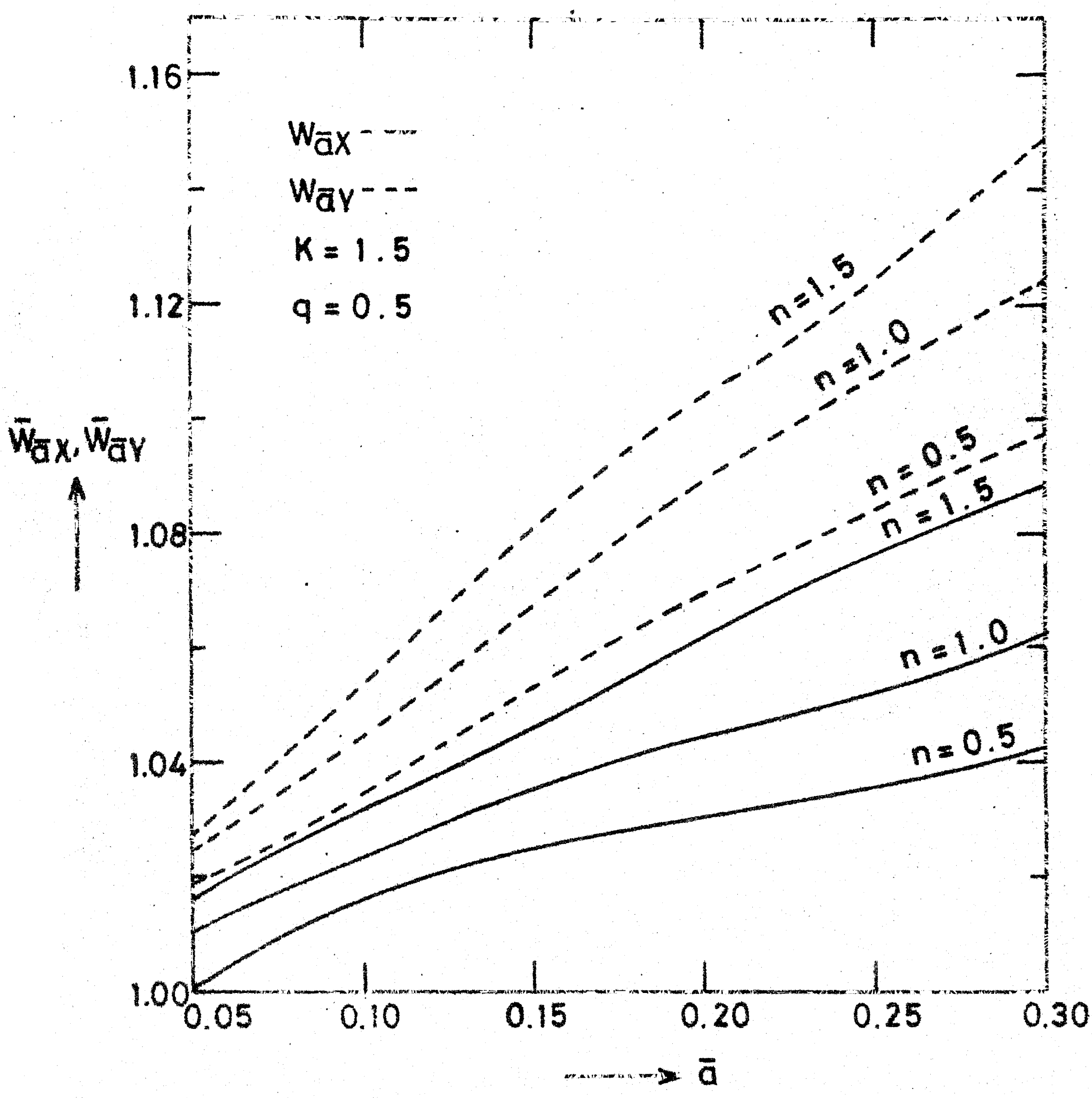

Fig. 2.11 Load capacity ratios $\bar{W}_{\bar{a}X}$ and $\bar{W}_{\bar{a}Y}$ vs. peripheral layer thickness $\bar{a}$ for various values of flow behaviour index n, K>1.

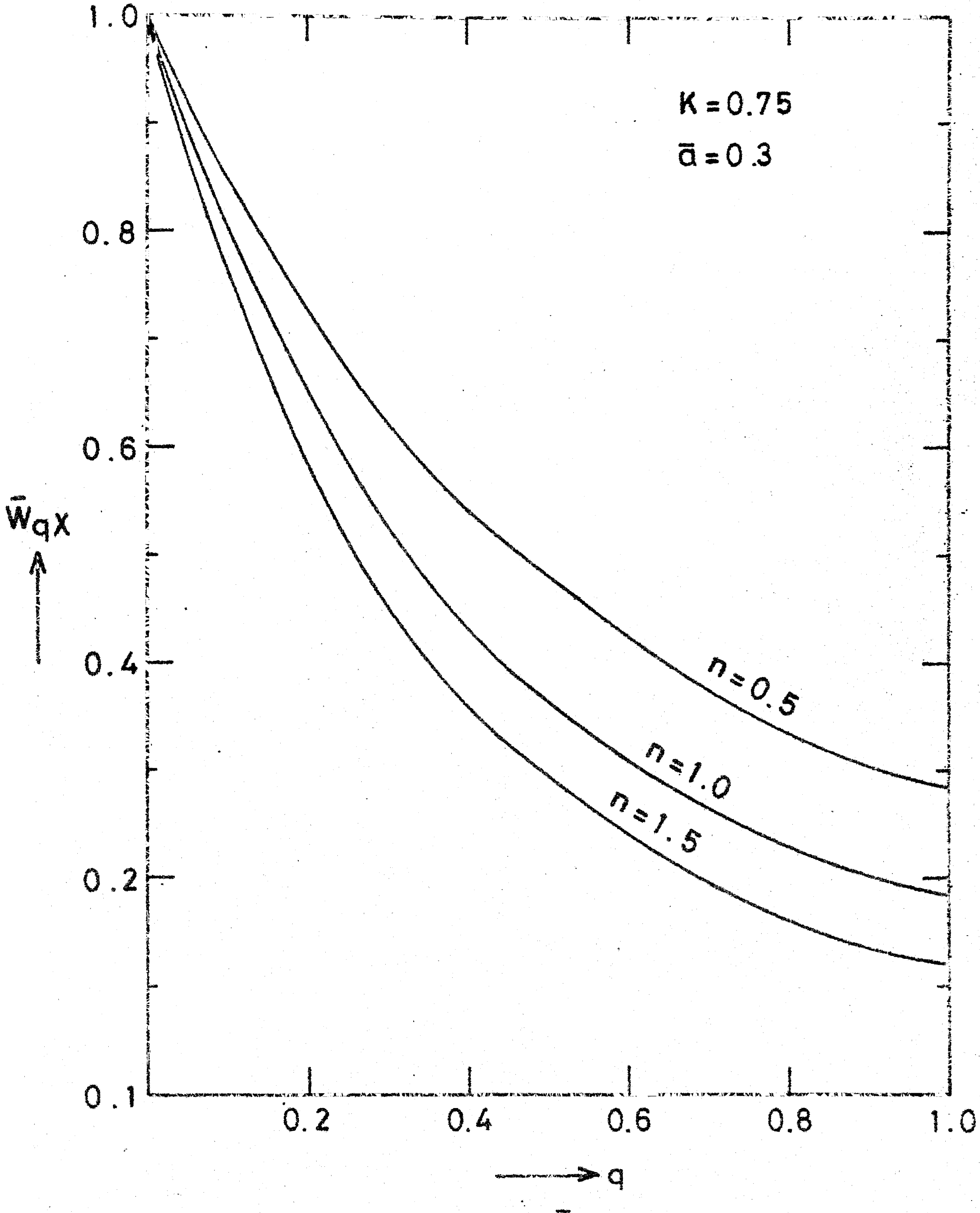

Fig. 2.12 Load capacity ratio $\bar{W}_q X$ vs. thermal factor q for various values of flow behaviour index n.

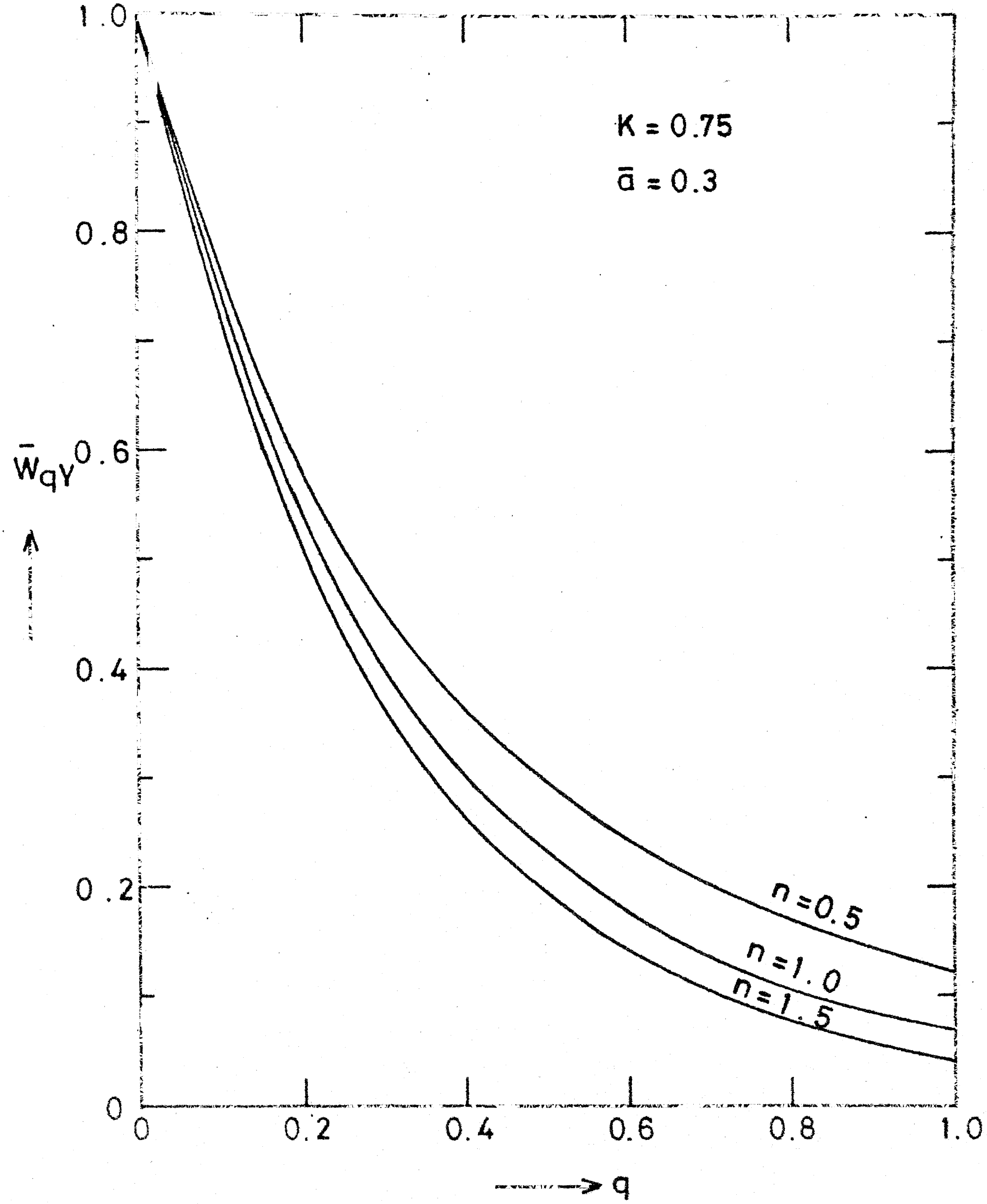

Fig. 2.13 Load capacity ratio $\bar{W}_{q\gamma}$ vs. thermal factor q for various values of flow behaviour index n.

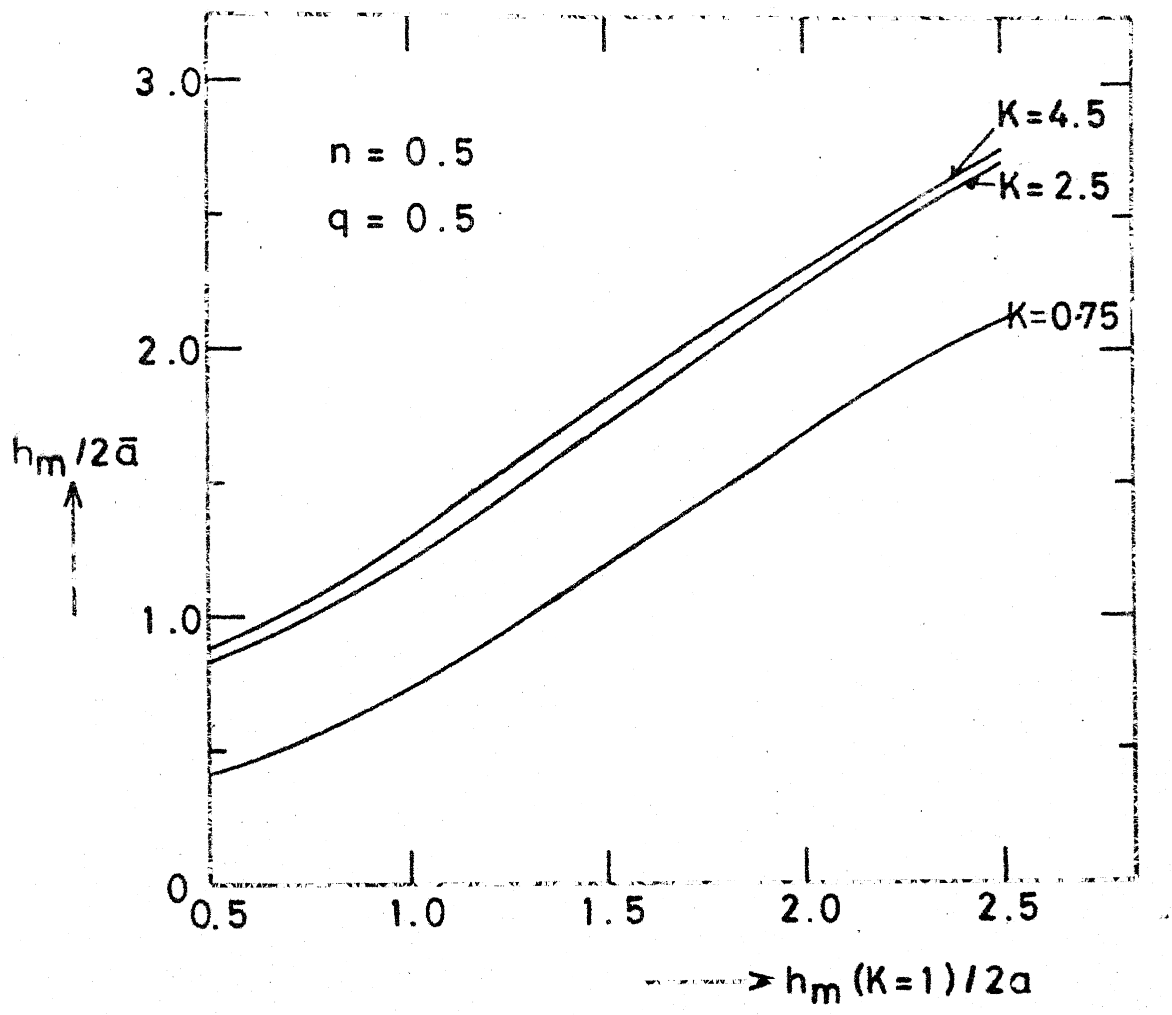

Fig. 2.14 Effect of consistency ratio K on ehd minimum film thickness h_m for various values of K.

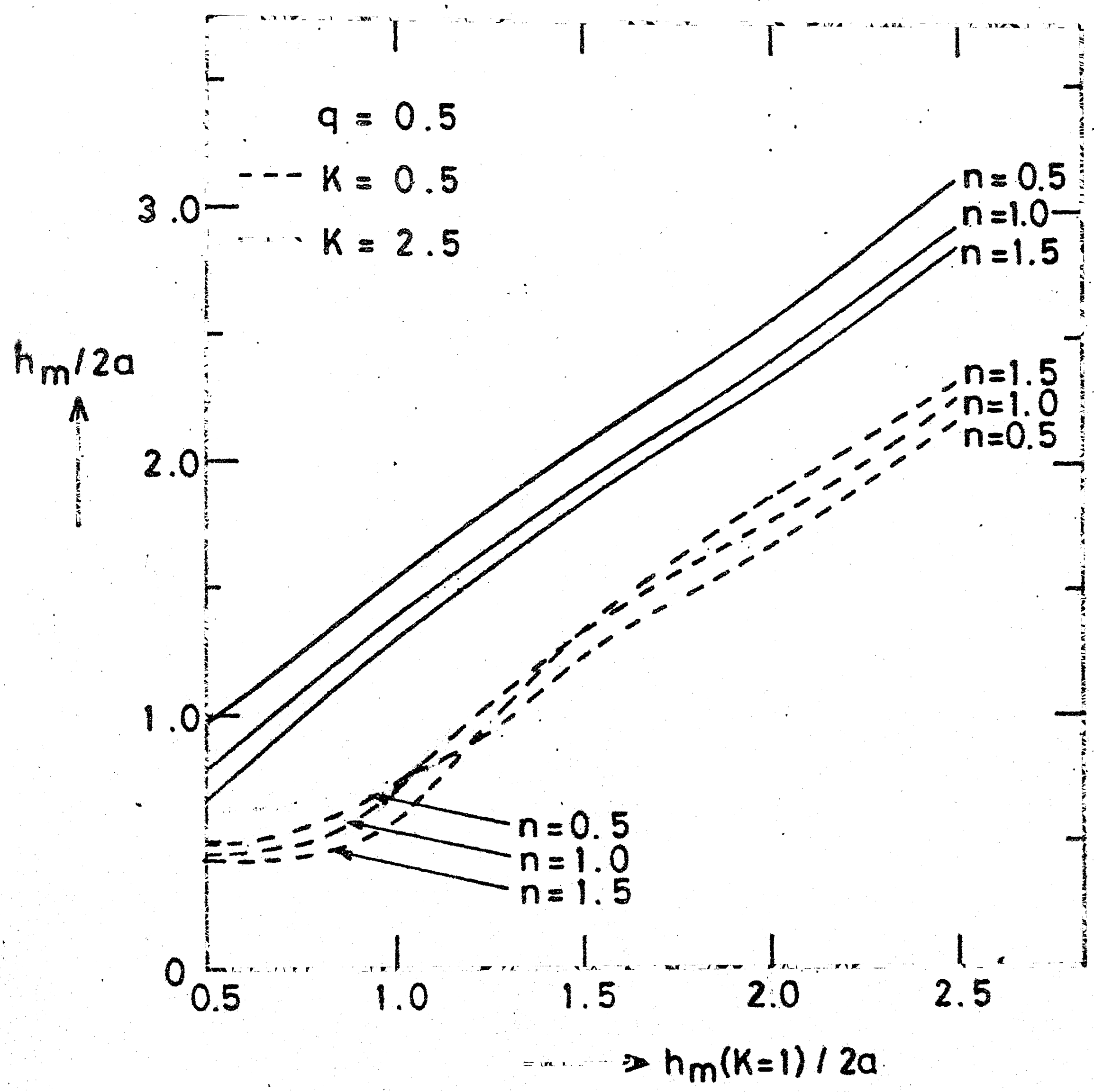

Fig. 2.15 Effect of consistency ratio K on ehd minimum film thickness h_m for values of n, K<1 and K>1.

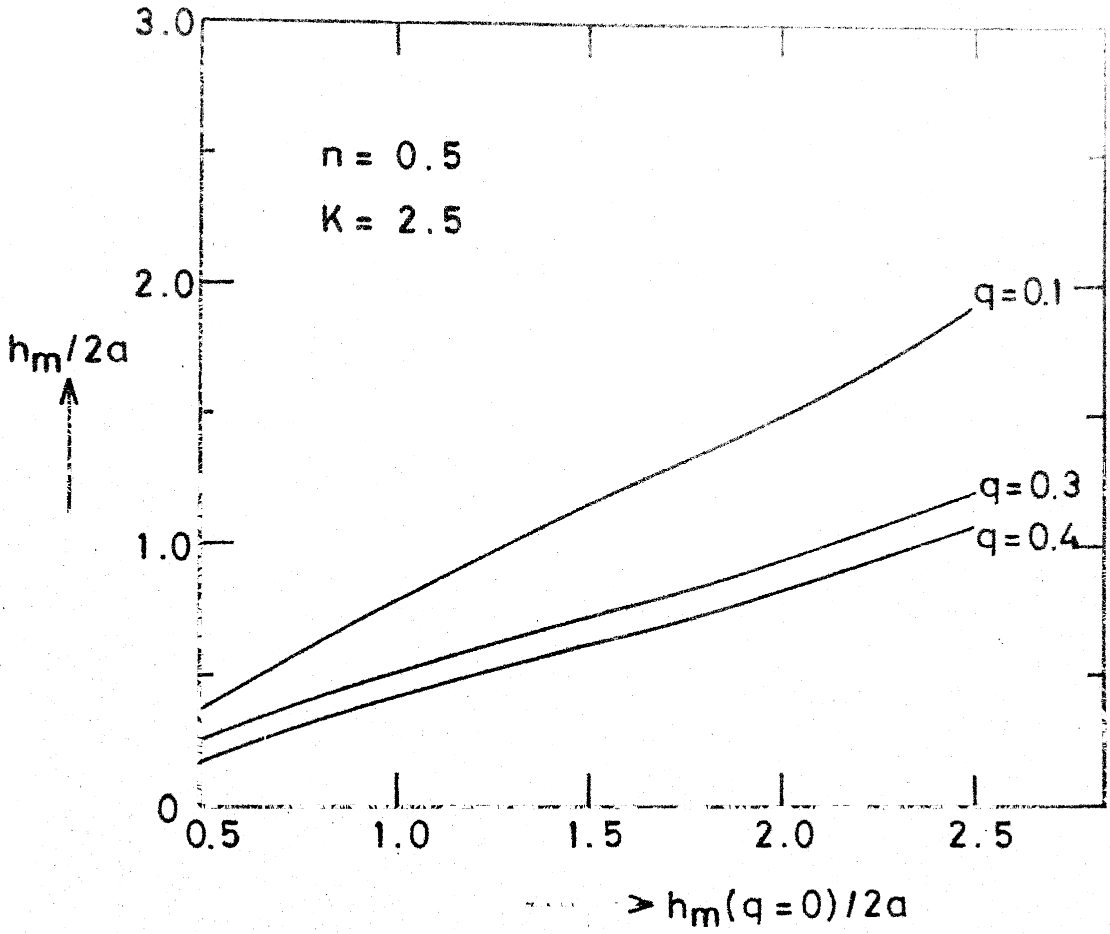

Fig. 2.16 Effect of thermal factor q on ehd minimum film thickness h_m for various values of q, K>1

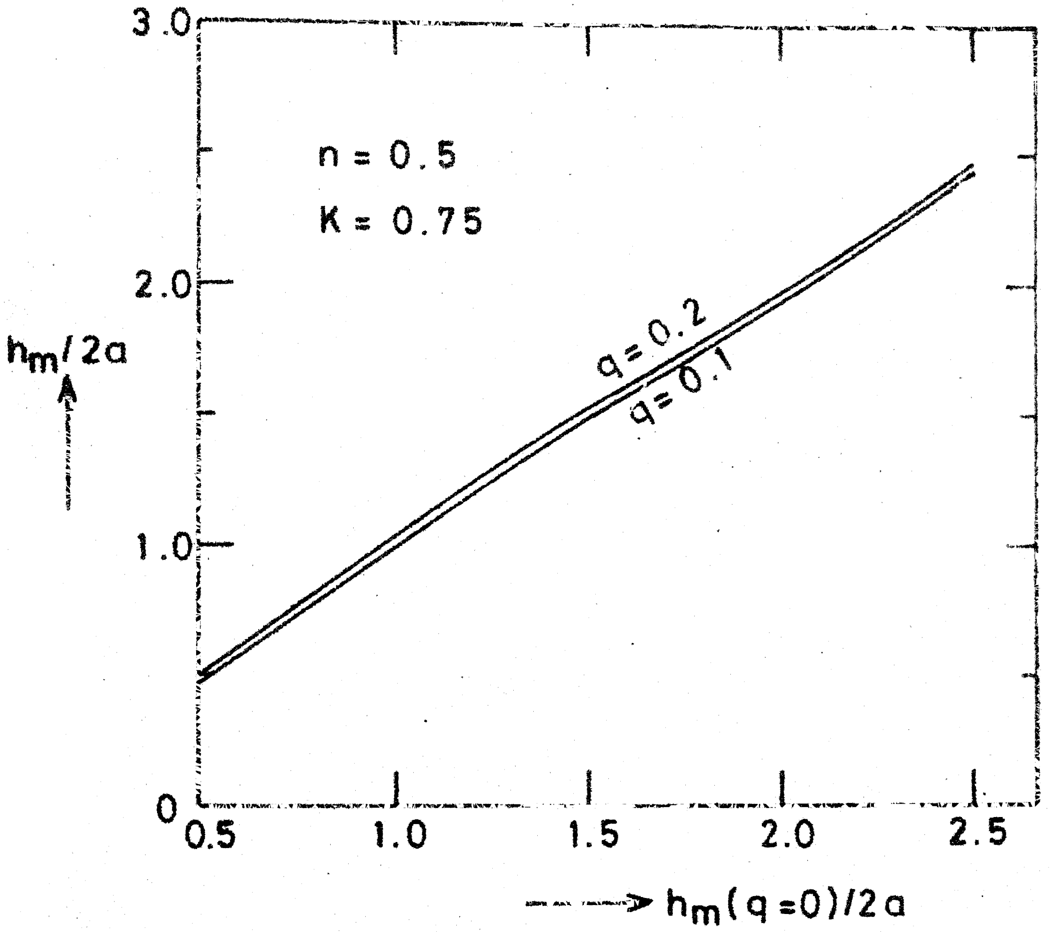

Fig. 2.17 Efect of thermal factor q on ehd minimum film thickness h_m for different values of $q, K<1$.

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NOMENCLATURE

a	peripheral layer thickness
$\bar{a}$	non-dimensional quantity correspond to a
d	half width of the Hertzian contact
d_1	point on the inlet region where $h = 2a$
E	reduced modulus of the two cylinders
E_1	Young's modulus of the cylinders
f_1, f_2	frictional forces at $y = -h/2$ and $y = h/2$ respectively
h	film thickness
h_o	minimum film thickness in the hydrodynamic lubrication
h_1	inlet film thickness
h^*	film thickness at the point where pressure gradient is zero
h_m	minimum film thickness in the ehd lubrication
H, H_1, H	non-dimensional quantities corresponding to h, h_1 and h respectively
K	consistency ratio
m, m_o, m_1	consistency indices
n	flow behaviour index
p	hydrodynamic pressure
p_1, p_2	pressure defined in the regions with positive and negative pressure gradients
p_E	pressure in the ehd lubrication
p_m	pressure at $x = -d_1$ (eqn. 2.73)

p_{1E}, p_{2E}	ehd pressures in the regions defined in eqns. (2.69) and (2.70) respectively
r	radius of the roller
r_E	radius of the roller in the ehd lubrication
R	radius of the equivalent roller
R_E	radius of the equivalent roller in the ehd lubrication
u, v	velocities along the coordinate axes
U	rolling velocity
V	normal velocity
$V_{h/2}$	resultant normal velocity at $y = h/2$
W_x, W_y	x and y components of load
$\bar{W}_{KX}, \bar{W}_{KY}, \bar{W}_{qX}, \bar{W}_{qY}$	non-dimensional load ratio parameters
x, y	coordinate axes
$-x^*$	point of maximum pressure
x_c or x^*	point of cavitation
$X, -X^*, X^*$	non-dimensional quantities corresponding to x, $-x^*$ and x^* respectively
X^*_{K1}, X^*_{qo}	non-dimensional points of cavitation for the cases $K = 1$ and $q = 0$ respectively
τ	shear stress
ν	Poisson ratio

CHAPTER -3

EFFECTS OF CONSISTENCY VARIATION OF POWER LAW LUBRICANTS IN SQUEEZE FILMS

3.1 INTRODUCTION

The study of squeeze films has been one of the subjects of interest in lubrication applications and rheological studies. However, such a study is not of great practical value, but it is representative of time dependent lubrication [1]. Theoretical work on squeeze films using a Newtonian fluid on various geometric configurations was reported by Archibald [2]. Several investigators have analysed squeeze film lubrication using non-Newtonian fluids [3-5]. These studies, in general, do not take into consideration variation of viscosity/consistency of the operating lubricant. Such a consideration is warranted by an increase in the effective viscosity, particularly in the lubricant layer very near to bearing surfaces. Needs [6] observed a predominant enhancement of viscosity during squeezing. To detect the influence of bounding surfaces on viscosity of the squeezed films, he conducted a series of experiments using two optically plane parallel circular plates approaching each other with a normal velocity, and measured film thickness down to 0.635E-03 mm. He noticed an increasing discrepancy between the measured and theoretical (as measured from classical Newtonian theory) intervals of time (response time).

for film thicknesses less than 0.127E-02 mm. The increase in the effective viscosity in such a thin film was attributed to the influence of metal surfaces on the fluid layer in their convicinity, causing the fluid to behave more rigid. Derjaguin [7] observed an increase in viscosity in the liquids where polar surface active substances were present at a distance upto 0.1μm from the solid boundary. He pointed out that in liquid polymers and in polymer solutions, there is generally an increase in viscosity at a distance upto 7 or 8 μm from solid boundary. Hayward and Isdale [8] attributed the abnormality in rheology to dirt and remarked that in pure liquids this would not happen. In the experimental findings of Askwith et.al. [9] it has been pointed out that the organic liquid in contact with the bearing surface formed a high viscous layer adjacent to it. Yousif et.al [10] observed that the molecules of the liquid in intimate contact with or to the adsorbed layer of the bounding surface must behave differently from the adjacent molecules in the bulk lubricant. However, the entire nature of the surface of the influence and the distance to which it penetrates has not been fully understood.

It is evident, on the whole, that there exists a strong case for the consideration of consistency variation in

squeeze films. The effects of consistency variation have been considered in Newtonian fluids [11-14]. Scant attention has been given to such a variation using non-Newtonian fluids [15]. With particular reference to squeeze films, Shukla et.al [15] analysed the non-Newtonian behaviour of lubricants through power law model by considering consistency variation across the film thickness. In this chapter, we consider the consistency variation across as well as along the film thickness by adopting the model we proposed for such a variation in Chapter 2.In particular, we study squeeze films on rigid solids for various geometries such as parallel plates, circular plates, roller and journal bearings etc. and the effect of consistency variation on load and response time is analysed in each case.-

3.2 PARALLEL PLATES

In this section, we consider the flow between two parallel plates of length 2d, approaching each other normally with a velocity V due to a symmetrically placed load (Fig.3.1). The plates are separated by a film thickness 2h. With the usual assumptions of lubrication theory, the governing eqns. of motion for a power law fluid in the case of squeezing is obtained

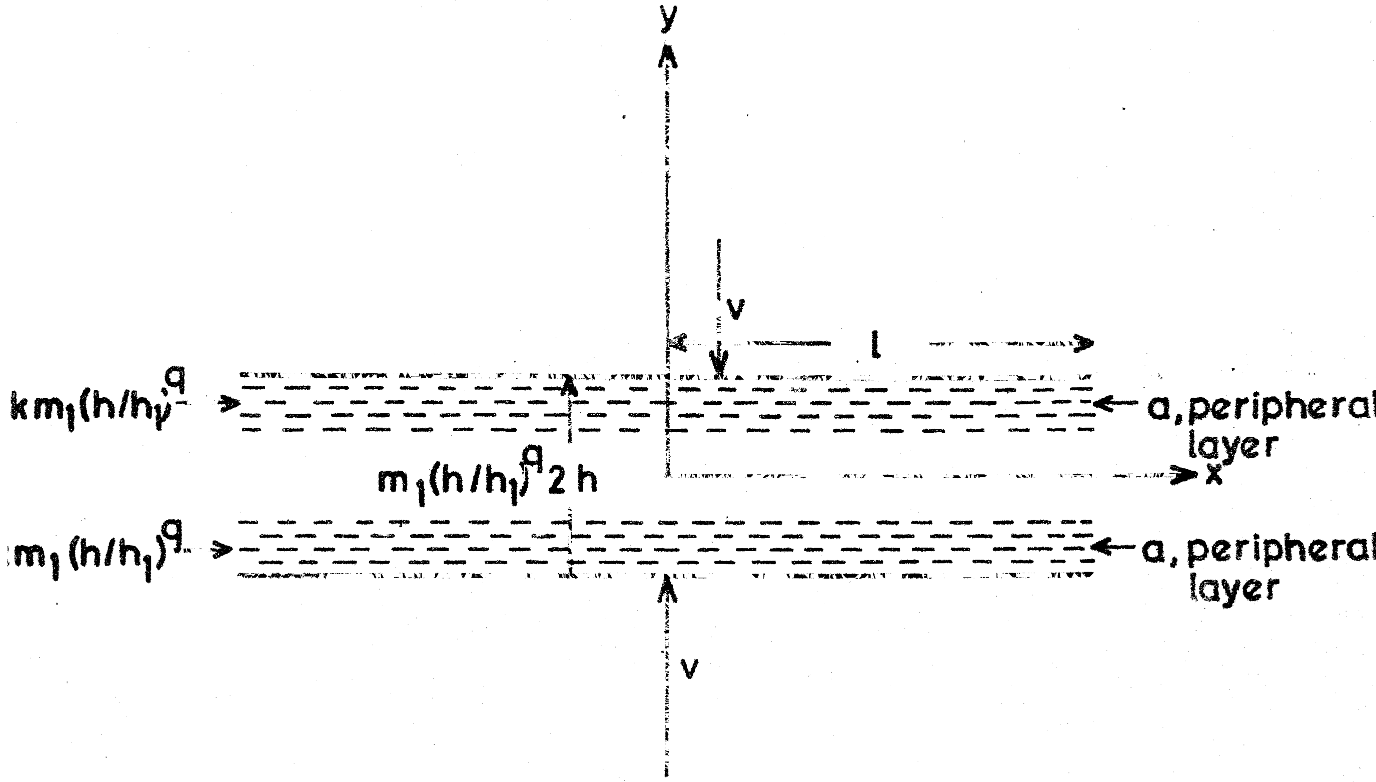

Fig. 3.1 Squeeze film between parallel plates.

with reference to Fig.3.1 and putting U = 0 in eqn. (2.18).

$$\frac{d}{dx}\left[-\frac{n}{2n+1}\, h_1^{q/n}\, (f_o)\, h^{(2n+1-q)/n} \left(-\frac{1}{m_1}\frac{dp}{dx}\right)^{1/n}\right] = V \qquad (3.1)$$

where

$$(f_o) = 1-(1-K^{-1/n})\,\{1-(1-a/h)^{(2n+1)/n}\} \qquad (3.2)$$

and h_1 is the initial film thickness measured at $x = -d$ just before squeezing commences.

Pressure attains its maximum at $x = 0$, i.e., $\frac{dp}{dx} = 0$ at $x = 0$. Using this condition in the integration of eqn. (3.1) we get,

$$\frac{dp}{dx} = -m_1\left(\frac{2n+1}{n}\,\frac{Vx}{(f_o)}\right)^n \left(\frac{1}{h_1}\right)^q \left(\frac{1}{h}\right)^{2n+1-q} \qquad (3.3)$$

Integrating again eqn. (3.3) using condition $p = 0$, at $x = d$, we obtain the expression for pressure p. Denoting it by $p_{K,q}$ we have

$$p_{K,q} = \frac{m_1}{n+1}\left(\frac{2n+1}{n}\,\frac{V}{(f_o)}\right)^n \left(\frac{1}{h_1}\right)^q \left(\frac{1}{h}\right)^{2n+1-q} (d^{n+1}-x^{n+1}) \qquad (3.4)$$

The load capacity $W_{K,q}$ per unit width is given by

$$W_{K,q} = 2\int_0^d p_{K,q}(x)\,dx \qquad (3.5)$$

which on using eqn. (3.4) becomes

$$W_{K,q} = \frac{2m_1}{n+2}\left(\frac{2n+1}{n}\,\frac{V}{(f_o)}\right)^n d^{n+2} \left(\frac{1}{h_1}\right)^q \left(\frac{1}{h}\right)^{2n+1-q} \qquad (3.6)$$

The squeezing time $t_{K,q}$ from an initial film thickness $2h_1$ to a subsequent film thickness $2h_2$, say, is obtained by putting $-V = \frac{dh}{dt}$ in eqn. (3.6) and integrating, we have

$$t_{K,q} = \frac{2n+1}{n} d^{(n+2)/n} \left(\frac{2m_1}{(n+2)W_{K,q}}\right)^{1/n} \left(\frac{1}{h_1}\right)^{q/n} \int_{h_2}^{h_1} \frac{dh}{(f_o)h^{(2n+1-q)/n}} \tag{3.7}$$

Considering consistency variation along the film thickness only (which can be obtained by taking K=1), eqns. (3.6) and (3.7) become

$$W_{1,q} = \frac{2m_1}{n+2}\left(\frac{2n+1}{n}V\right)^n d^{n+2} \left(\frac{1}{h_1}\right)^q \left(\frac{1}{h}\right)^{2n+1-q} \tag{3.8}$$

and

$$t_{1,q} = \frac{2n+1}{n+1-q} d^{(n+2)/n} \left(\frac{2m_1}{n+2\ W_{1,q}}\right)^{1/n} \left(\frac{1}{h_1}\right)^{q/n} \times$$

$$\times \left[\left(\frac{1}{h_2}\right)^{(n+1-q)/n} - \left(\frac{1}{h_1}\right)^{(n+1-q)/n} \right] \tag{3.9}$$

To determine the effect of thermal factor q on load and response time, we define the following quantities :

$$\bar{W}_q = \frac{W_{K,q}}{W_{K,o}} = H^q \tag{3.10}$$

$$\bar{t}_q = \frac{t_{K,q}}{t_{K,o}} = \frac{I_{K,q}}{I_{K,o}} \tag{3.11}$$

where

$$I_{K,q} = \int_{H_2}^{1} 1/[(F_o)H^{(2n+1-q)/n}]dH \tag{3.12}$$

$$(F_o) = 1-(1-K^{-1/n})\{1-(1-\bar{a}/H)^{(2n+1)/n}\} \tag{3.13}$$

$$H = h/h_1,\ \bar{a} = a/h_1,\ H_2 = h_2/h_1 \tag{3.14}$$

To study the effect of consistency ratio K on load and time of squeezing, we assume that the peripheral layer thickness varies with film thickness only, i.e., $a = \delta^* h$ where $\delta^* \leq 1$ is a constant, and define quantities $\bar{W}_K$ and $\bar{t}_K$ from eqns. (3.6)-(3.9) as follows :

$$\bar{W}_K = \frac{W_{K,q}}{W_{1,q}} = \left(\frac{1}{F_{\delta^*}}\right)^n \tag{3.15}$$

and

$$\bar{t}_K = \frac{t_{K,q}}{t_{1,q}} = \frac{1}{F_{\delta^*}} \tag{3.16}$$

where $\quad F_{\delta^*} = 1-(1-K^{-1/n})\{1-(1-\delta^*)^{(2n+1)/n}\}$ (3.17)

Eqn. (3.10) gives the variation of load ratio $\bar{W}_q$ with q. Since $\bar{W}_q$ is a function of t and q and at any instant of time during squeezing $H < 1$, thus, $\bar{W}_q$ decreases as q increases. This is in conformity with the established result that the thermal effects tend to decrease the pressure generation and hence the load capacity. Eqn. (3.11) gives the variation of $\bar{t}_q$ with q for various values of n (Fig.3.2). As q increases the response time decreases for any fixed but arbitrary n, possibly due to the thinning of lubricant with thermal effects. These effects significantly alter response time for pseudoplastics compared to dilatants.

To study the effects of K and $\bar{a}$ on $\bar{W}_K$ and $\bar{t}_K$, eqns. (3.15) and (3.16) are evaluated. The effect of K on $\bar{W}_K$ is studied

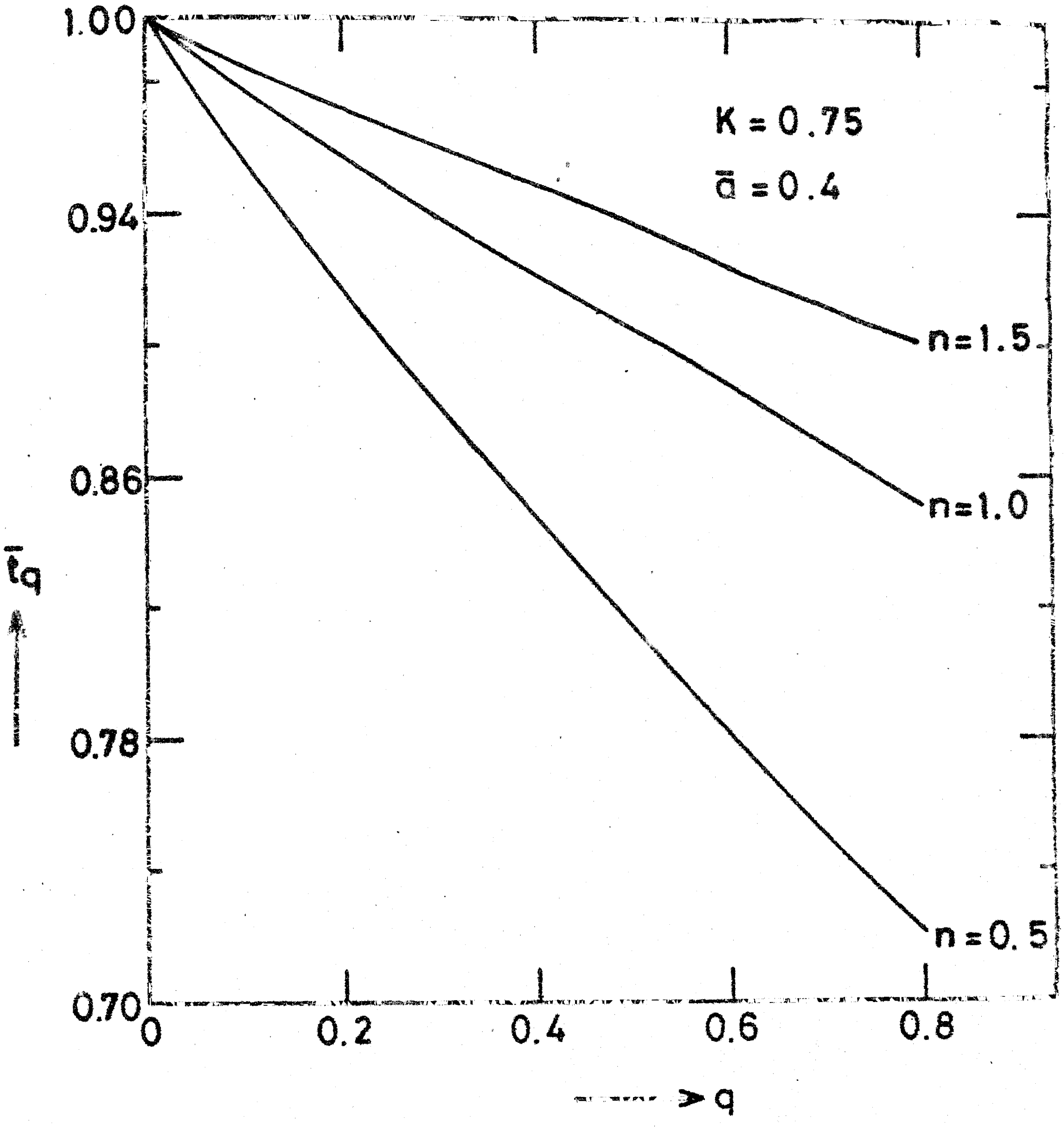

Fig. 3.2 Variation of response time ratio $\bar{t}_q$ with thermal factor q for parallel plates.

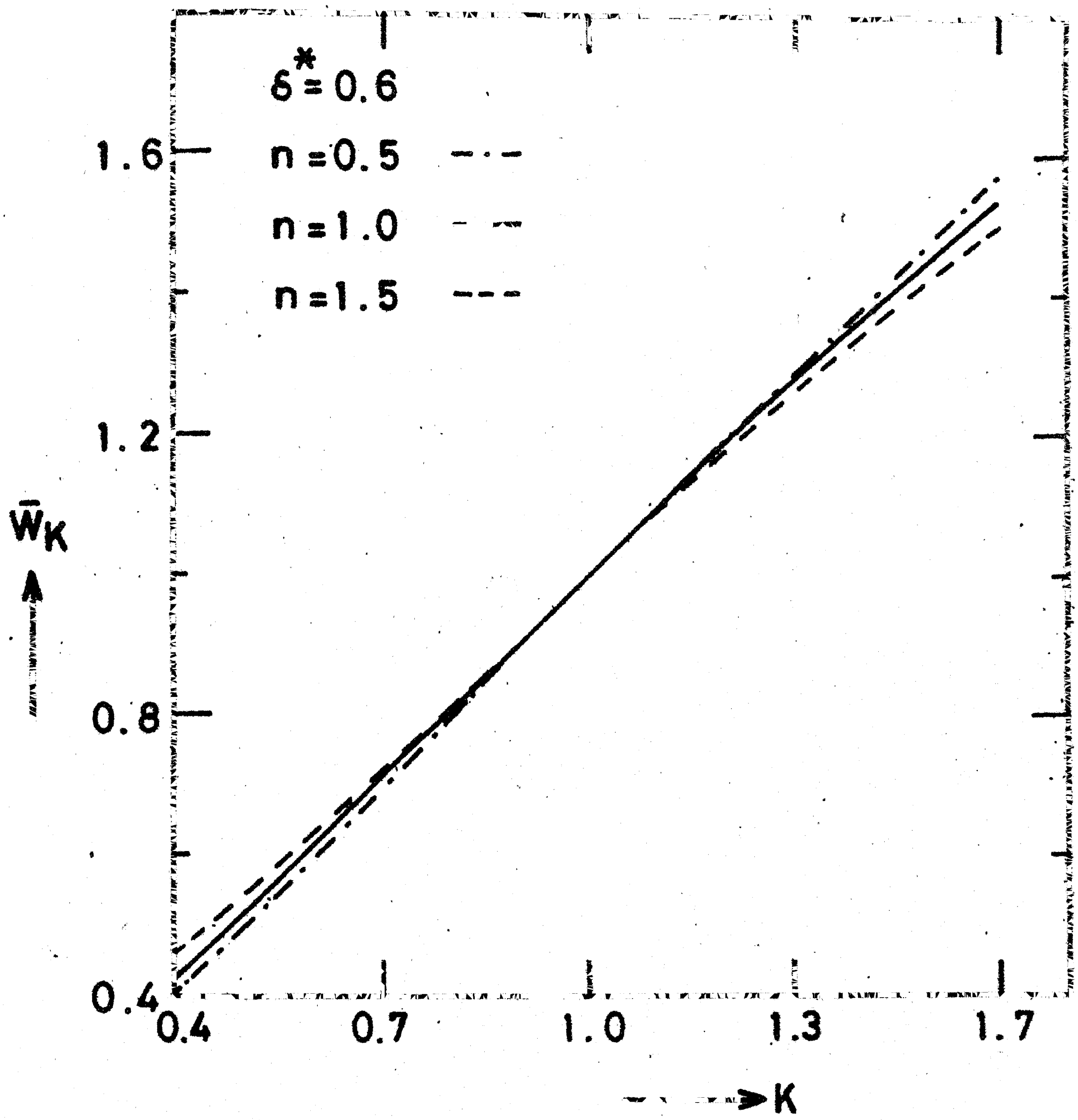

Fig. 3.3 Effect of consistency ratio K on load ratio $\bar{W}_K$ for parallel plates.

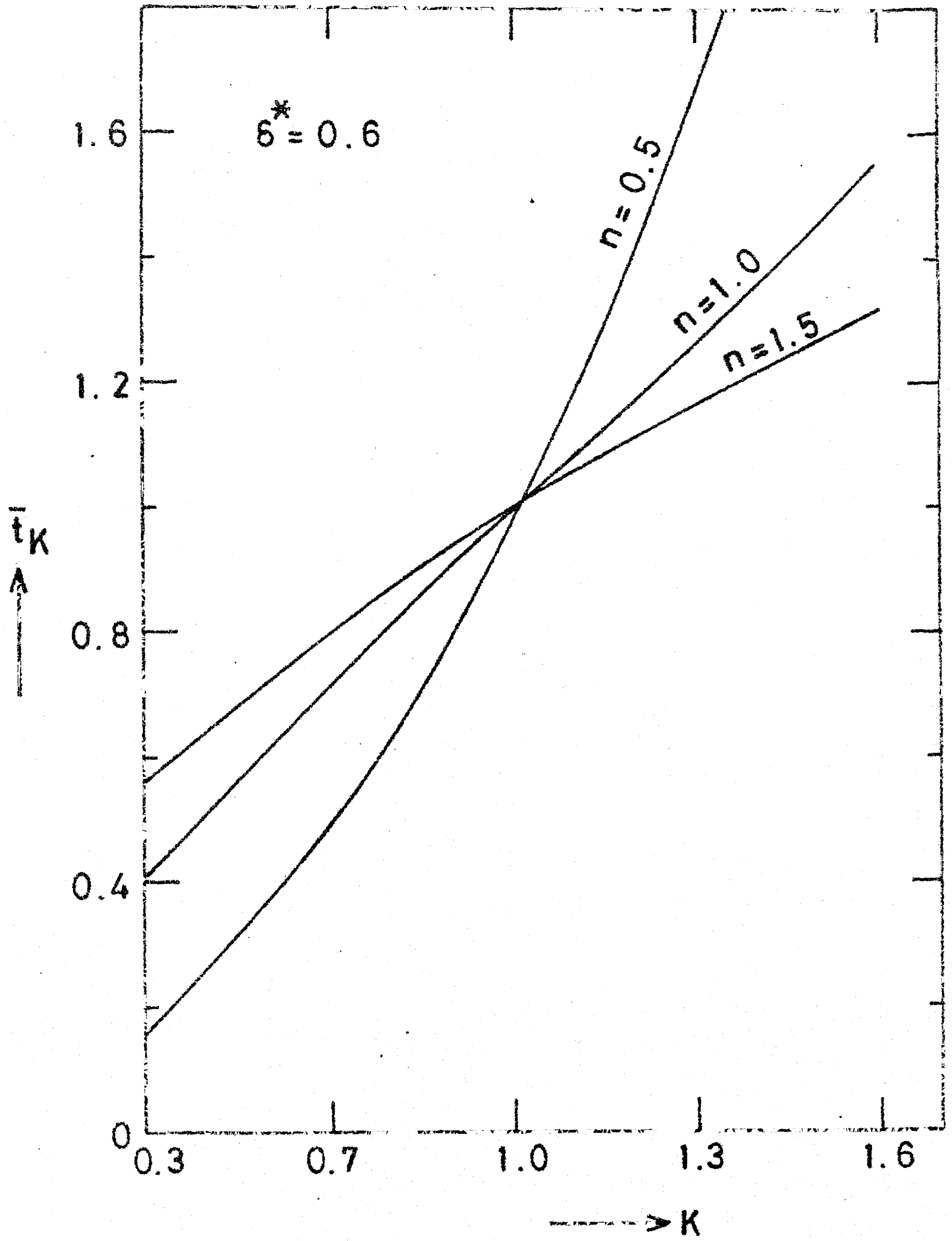

Fig. 3.4 Variatio of response time ratio $\bar{t}_K$ with consistency ratio K for parallel plates.

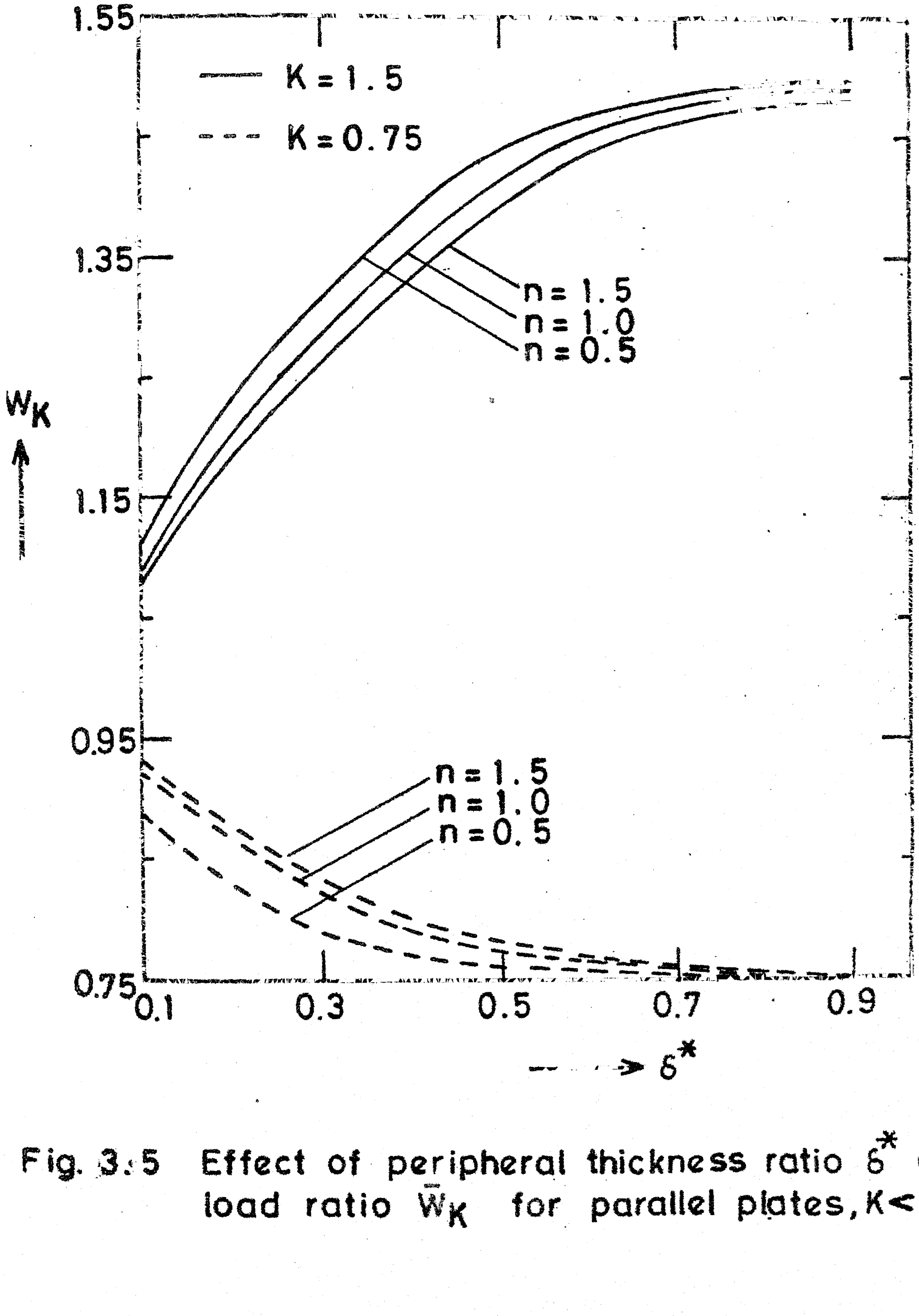

Fig. 3.5 Effect of peripheral thickness ratio δ^* on the load ratio $\bar{W}_K$ for parallel plates, K<1 and K>1.

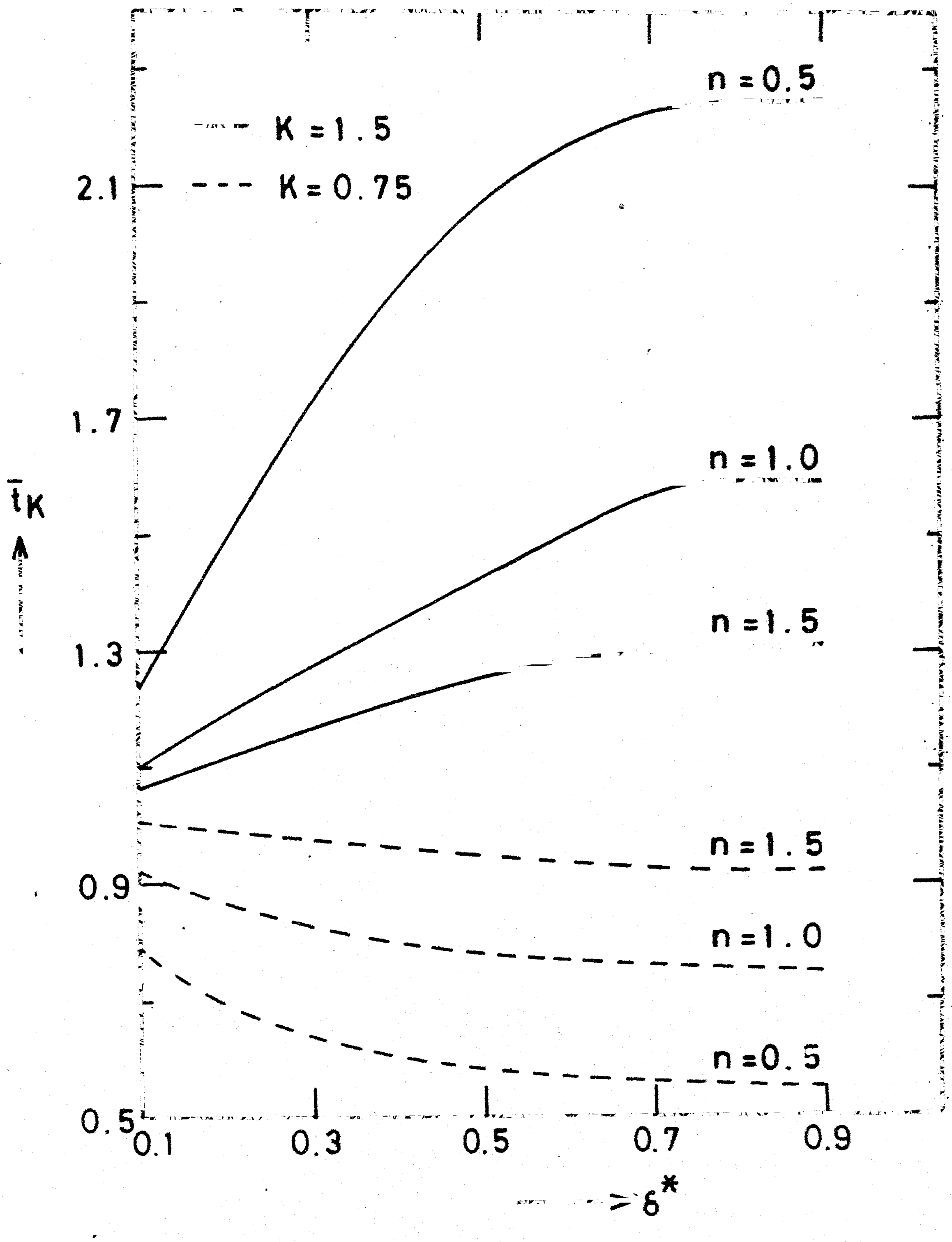

Fig. 3.6 Variation of response time ratio $\bar{t}_K$ with peripheral thickness ratio δ^* for parallel plates, $K<1$ and $K>1$.

in Fig.3.3 where the peripheral layer thickness a varies in constant proportion to half of film thickness 2h. The effect of an increase in consistency in peripheral region is to increase load capacity for all n. Further, load capacity is more altered for pseudoplastics compared to dilatants. From Fig.3.4 we note that the change in K alters the response time more for pseudoplastics than for dilatants.

The effect of K on load and response time is further eloborated in Figs. 3.5 and 3.6 respectively for specified value of h. For fixed h, increase in δ^* means an increase in peripheral thickness and load capacity and response time increase or decrease according as $K \gtrless 1$. Further, for specified film thickness and K, load capacity and response time are altered more for pseudoplastics compared to dilatants due to variations in peripheral layer thickness.

3.3 PARALLEL CIRCULAR PLATES

We consider squeezing between two circular parallel plates each of radius R separated by a film thickness 2h (Fig.3.7). The plates approach each other with a normal velocity V symmetrically. The Reynolds equation for the flow of power law lubricant in the radial direction r can be obtained as

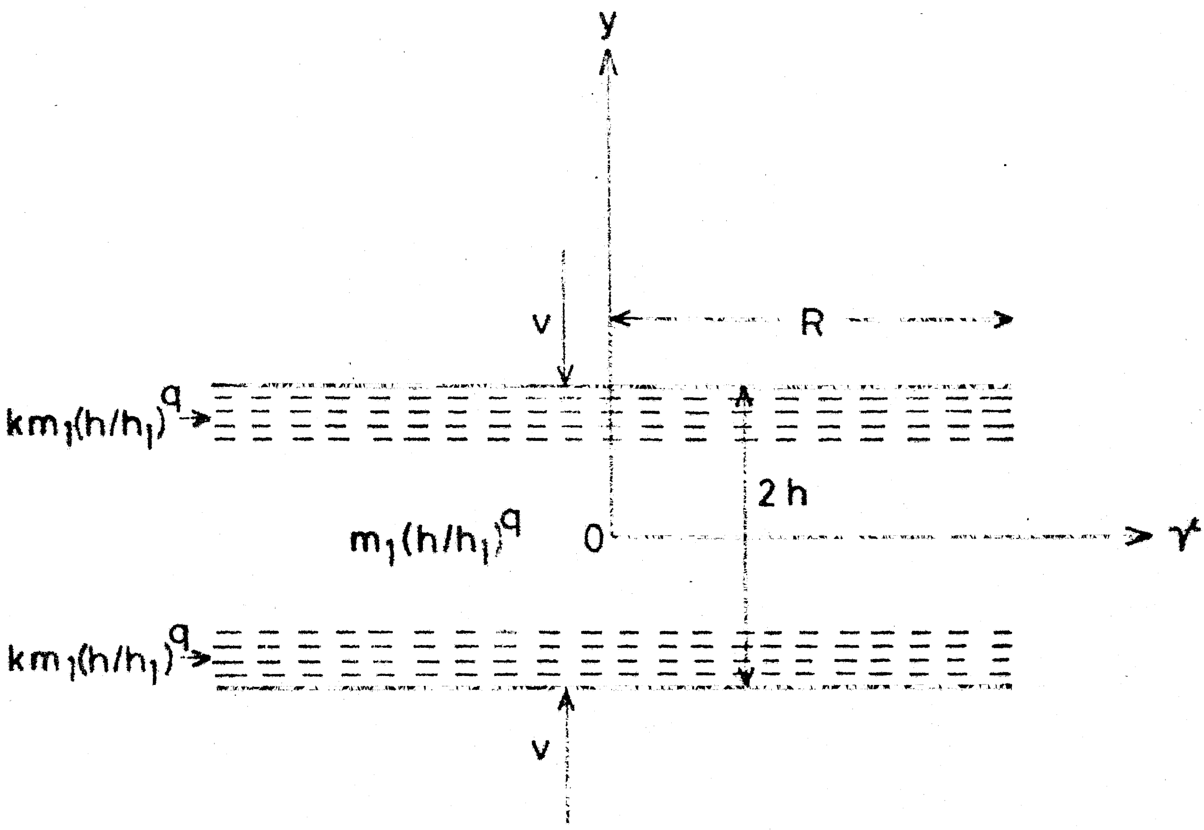

Fig. 3.7 Squeeze film between parallel circular plates.

$$\frac{1}{r}\frac{d}{dr}\left[\frac{n}{2n+1}\, h_1^{q/n}\,(f_o)\, h^{(2n+1-q)/n}\, r\left(-\frac{1}{m_1}\frac{dp}{dr}\right)^{1/n}\right]=V \tag{3.18}$$

where (f_o) is defined in eqn. (3.2).

Using the boundary conditions

$$\frac{dp}{dr} = 0 \text{ at } r = 0 \tag{3.19}$$

and $\qquad p = 0 \quad \text{at } r = R$

we can obtain an expression for pressure by integrating twice, eqn. (3.18). Denoting it by $p_{K,q}$ we have

$$p_{K,q} = \frac{m_1}{n+1}\left(\frac{2n+1}{2n}\,\frac{V}{(f_o)}\right)^n \left(\frac{1}{h_1}\right)^q \left(\frac{1}{h}\right)^{2n+1-q} (R^{n+1}-r^{n+1})$$

The load capacity $W_{p,K}$ is given by

$$W_{K,q} = \int_o^R 2\pi\, r\, p\, dr \tag{3.21}$$

which on using eqn. (3.20) yields,

$$W_{K,q} = \frac{\pi m_1}{n+3}\left(\frac{2n+1}{2n}\,\frac{V}{(f_o)}\right)^n R^{n+3}\left(\frac{1}{h_1}\right)^q \frac{1}{h^{2n+1-q}} \tag{3.22}$$

The time of squeezing $t_{K,q}$ to reduce the initial film thickness $2h_1$ to a subsequent film thickness $2h_2$ is given by

$$t_{K,q} = \frac{2n+1}{2n}\left(\frac{\pi m_1}{n+3}\,\frac{1}{W_{K,q}}\right)^{1/n} R^{(n+3)/n}\left(\frac{1}{h_1}\right)^{q/n} \times$$

$$\times \int_{h_2}^{h_1} \frac{1}{(f_o)h^{(2n+1-q)/n}}\, dh \tag{3.23}$$

As in the case of parallel plates, the quantities to study the effects of q and K on load and time of squeezing can be defined as

$$\bar{W}_q = \frac{W_{K,q}}{W_{K,0}} = H^q \tag{3.24}$$

$$\bar{t}_q = \frac{t_{K,q}}{t_{K,0}} = \frac{I_{K,q}}{I_{K,o}} \tag{3.25}$$

and

$$\bar{W}_K = \frac{W_{K,q}}{\bar{W}_{1,q}} = \left(\frac{1}{F_\delta *}\right)^n \tag{3.26}$$

$$\bar{t}_K = \frac{t_{K,q}}{t_{1,q}} = \frac{1}{F_\delta *}$$

where $I_{K,q}$ and $F_\delta *$ are defined in eqns. (3.12) and (3.17) respectively.

It is seen that the expressions for bearing characteristics obtained for parallel circular plates (eqns. (3.24)-(3.27)) are the same as those obtained for parallel plates (eqns. (3.10), (3.11), (3.15) and (3.16)). Hence, similar interpretations can be made in this case as well.

3.4 ROLLER BEARINGS

The problem considered is that of the flow of a power law lubricant between two identical rollers each of radius r which approach each other with a velocity V (Fig. 3.8). The rollers have a projected length of fluid film 2d and are separated by a film thickness 2h. The film thickness is small compared with the radius of the rollers. With the

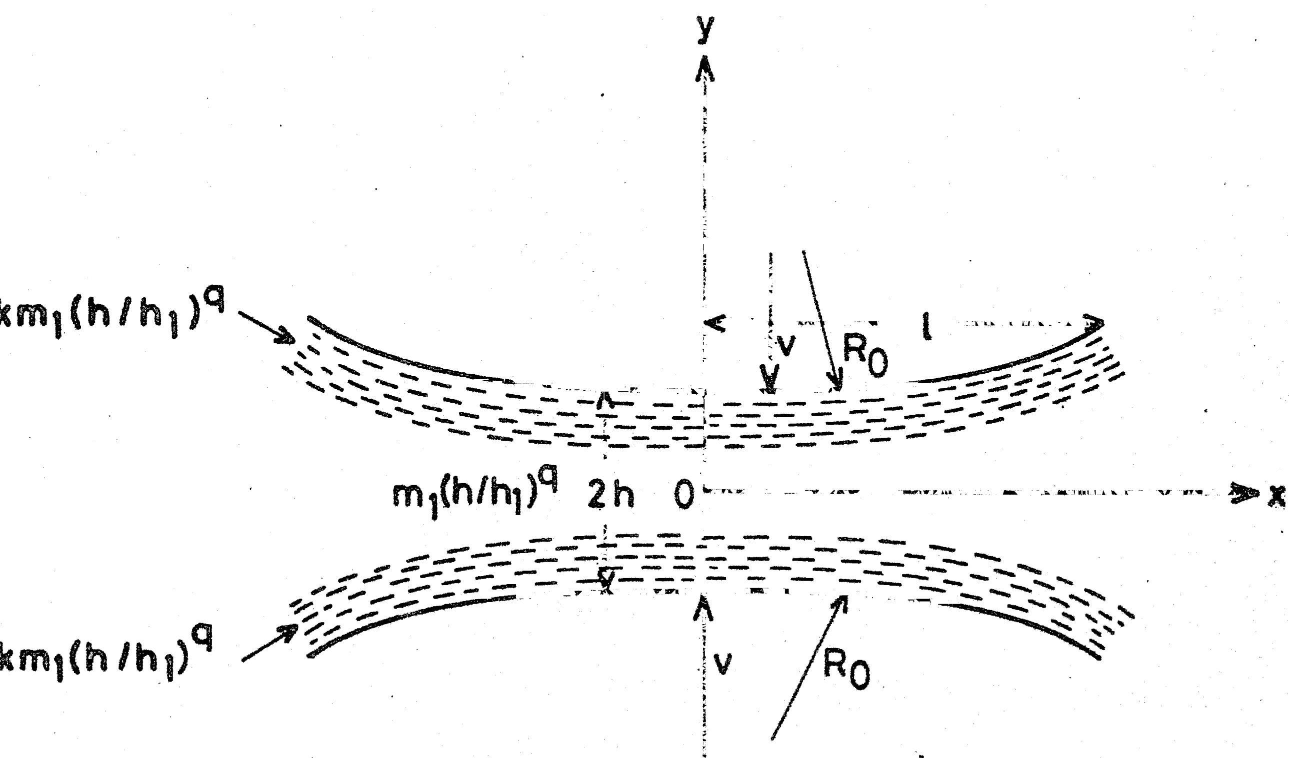

Fig.3.8 Squeezing between two rolling surfaces.

usual assumption of lubrication theory and symmetry of bearing configuration, the Reynolds equation can be written in the case of squeezing for rollers using eqn. (2.18) and Fig.3.8:

$$\frac{d}{dx}\left[-\frac{n}{2n+1}\, h_1^{q/n}\, h^{(2n+1-q)/n}\, (f_o)\left(-\frac{1}{m}\frac{dp}{dx}\right)^{1/n}\right] = V \qquad (3.28)$$

where $2h_1$ is the initial film thickness measured at $x = d$ and h is given by

$$h = h_o + x^2/(2R) \qquad (3.29)$$

$2h_o$ being the minimum film thickness and $R = r/2$ the radius of the equivalent roller.

Because of symmetry of pressure profile about $x = o$, we have $\frac{dp}{dx} = o$ at $x = o$. Using this condition in the integration of eqn. (3.28) we get,

$$\frac{dp}{dx} = -m_1\left(\frac{2n+1}{n}\,\frac{Vx}{(f_o)}\right)^n \left(\frac{1}{h_1}\right)^q \left(\frac{1}{h}\right)^{2n+1-q} \qquad (3.30)$$

Integrating eqn. (3.30) with condition $p = o$ at $x = d$, we obtain the expression for pressure. Denoting it by $p_{K,q}$ we have

$$p_{K,q} = -m_1\left(\frac{2n+1}{n}\,V\right)^n \left(\frac{1}{h_1}\right)^q \int_x^d \frac{x^n}{(f_o)^n\, h^{2n+1-q}}\,dx \qquad (3.31)$$

The load component $W_{K,q}$ is given by

$$W_{K,q} = 2\int_o^d p_{K,q}(x)\,dx \qquad (3.32)$$

which on using eqn. (3.31) simplifies to

$$W_{K,q} = 2m_1 \left(\frac{2n+1}{n} V\right)^n \left(\frac{1}{h_1}\right)^q \int_0^d \frac{x^{n+1}}{(f_o)^n \, h^{2n+1-q}} \, dx \tag{3.33}$$

The time of squeezing $t_{K,q}$ from an initial film thickness $2h_1$ to a subsequent film thickness $2h_2$ is obtained by substituting $-V = \frac{dh}{dt}$ in eqn. (3.33) as

$$t_{K,q} = \frac{2n+1}{n} \left(\frac{2m_1}{W_{K,q}}\right)^{1/n} \left(\frac{1}{h_1}\right)^{q/n} \int_{h_2}^{h_1} \left[\int_0^d \frac{x^{n+1}}{(f_o)^n \, h^{2n+1-q}} \, dx \right]^{1/n} dh_o \tag{3.34}$$

To study the effects of consistency ratio K and thermal factor q on load as well as response time, we define the following quantities :

$$\bar{W}_K = \frac{W_{K,q}}{W_{1,q}} = \frac{I(\alpha_o)_{K,q}}{I(\alpha_o)_{1,q}} \tag{3.35}$$

$$\bar{t}_K = \frac{t_{K,q}}{t_{1,q}} = \frac{\int_{H_2}^{1} I^{1/n}(\alpha_1)_{K,q} \, dH_o}{\int_{H_2}^{1} I^{1/n}(\alpha_1)_{1,q} \, dH_o} \tag{3.36}$$

$$\bar{W}_q = \frac{W_{K,q}}{W_{K,o}} = H_o^q \, \frac{I(\alpha_o)_{K,q}}{I(\alpha_o)_{K,o}} \tag{3.37}$$

$$\bar{t}_q = \frac{t_{K,q}}{t_{K,o}} = \frac{\int_{H_2}^{1} I^{1/n}(\alpha_1)_{K,q} \, dH_o}{\int_{H_2}^{1} I^{1/n}(\alpha_1)_{K,o} \, dH_o} \tag{3.38}$$

where

$$I_{(\alpha_i)K,q} = \int_0^1 [X^{n+1}/\{(F_{(\alpha_i)})^n H_{(\alpha_i)}^{2n+1-q}\}]dX \quad i = 0,1, \tag{3.39}$$

$$(F_{(\alpha_i)}) = 1-(1-K^{-1/n})\{1-(1-\bar{a}_i/H_{(\alpha_i)})^{(2n+1)/n}\} \tag{3.40}$$

$$X = x/d,\ \bar{a}_i = a/h_i,\quad \alpha_i = d^2/2Rh_i \quad i = 1,2,\quad H_o = h_o/h_1$$

$$H_{(\alpha_o)} = 1 + \alpha_o X^2,\ H_{(\alpha_1)} = H_o + \alpha_1 X^2 \tag{3.41}$$

From Eqns. (3.35) and (3.36) $\bar{W}_K$ and $\bar{t}_K$ are calculated to determine the effect of K . They are represented against K in Figs. 3.9 and 3.10 respectively. Due to consistency variation across the film thickness, for $K < 1$ there is a decrease in the load ratio and response time for pseudoplastics relative to dilatants, and for $K > 1$ these characteristics increase for pseudoplastics compared to dilatants. Hence, we conclude that the change in consistency in the peripheral region alters the load and response time for pseudoplastics compared to dilatants. Similar behaviour is observed for $q = 0$ in the reference [15].

Figs. 3.11 and 3.12 depict the variation of load ratio and response time ratio with factors α_o and α_1 for various values of n. For fixed value of h_o and h_1, α_o and α_1 can be interpreted to represent the variation of peripheral layer thickness. Thus, we observe from

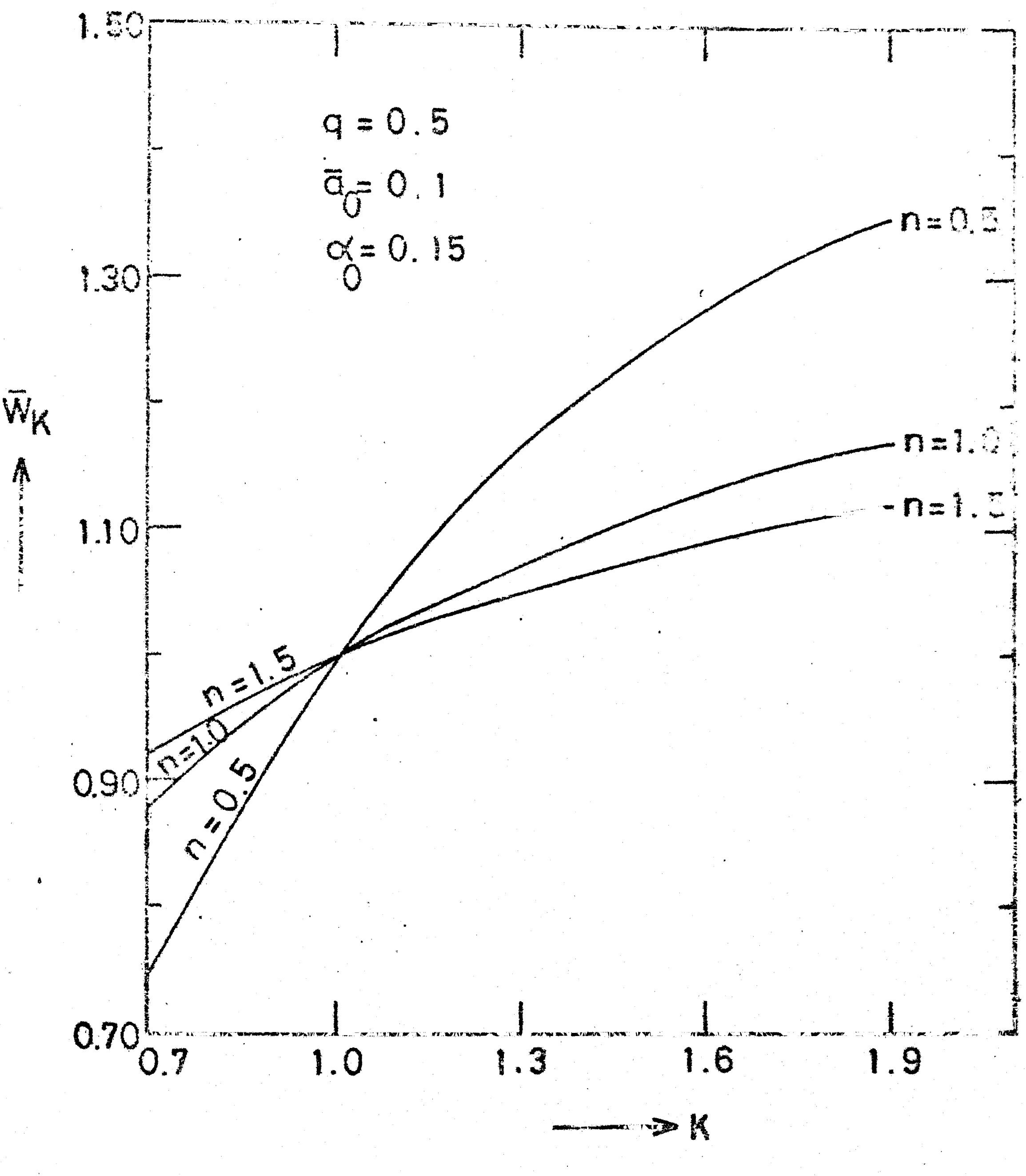

Fig. 3.9 Effect of consistency ratio K on load ratio parameter $\bar{W}_K$ for roller bearing.

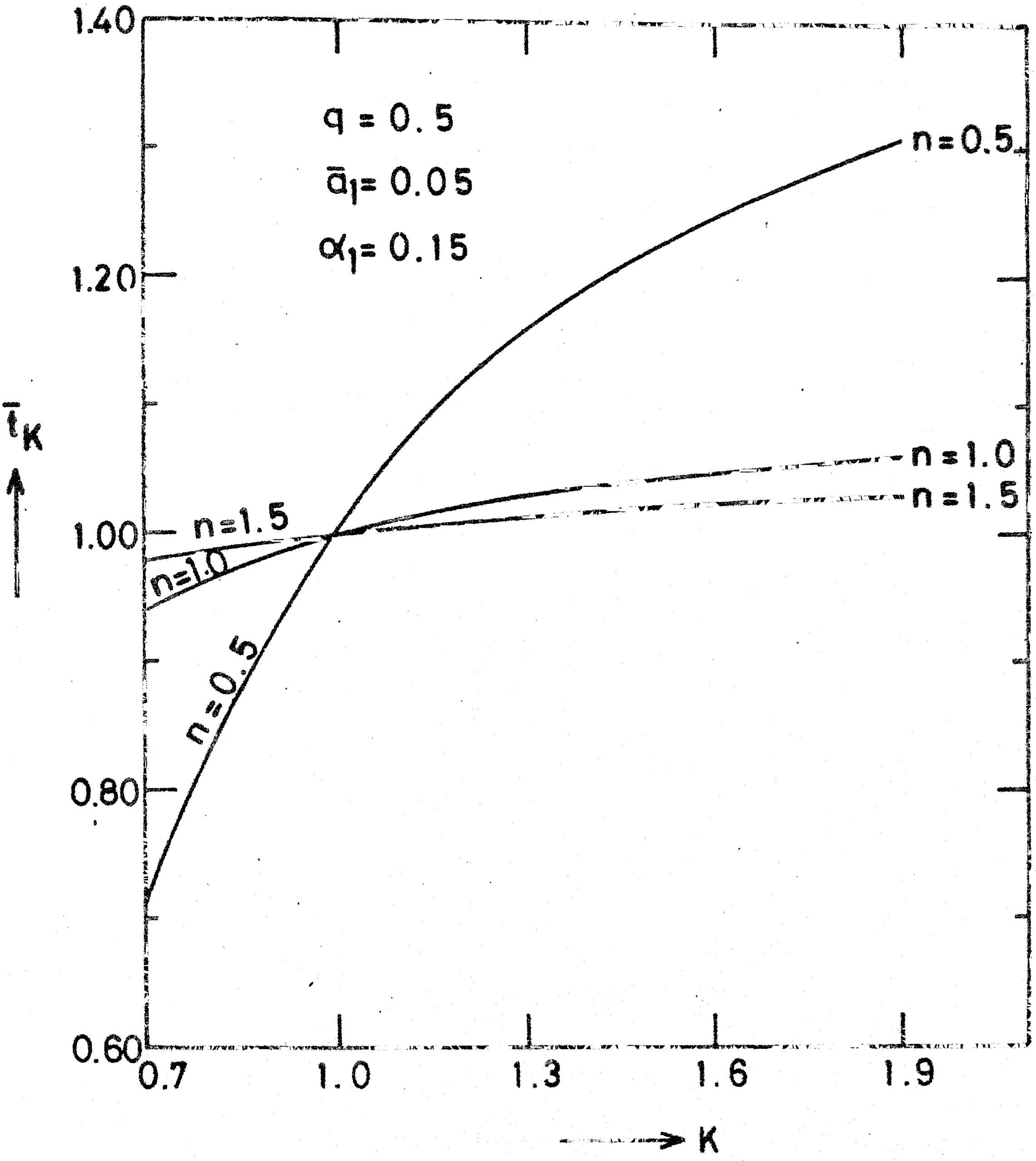

Fig. 3.10 Variation of response time ratio $\bar{t}_K$ with consistency ratio K for roller bearing.

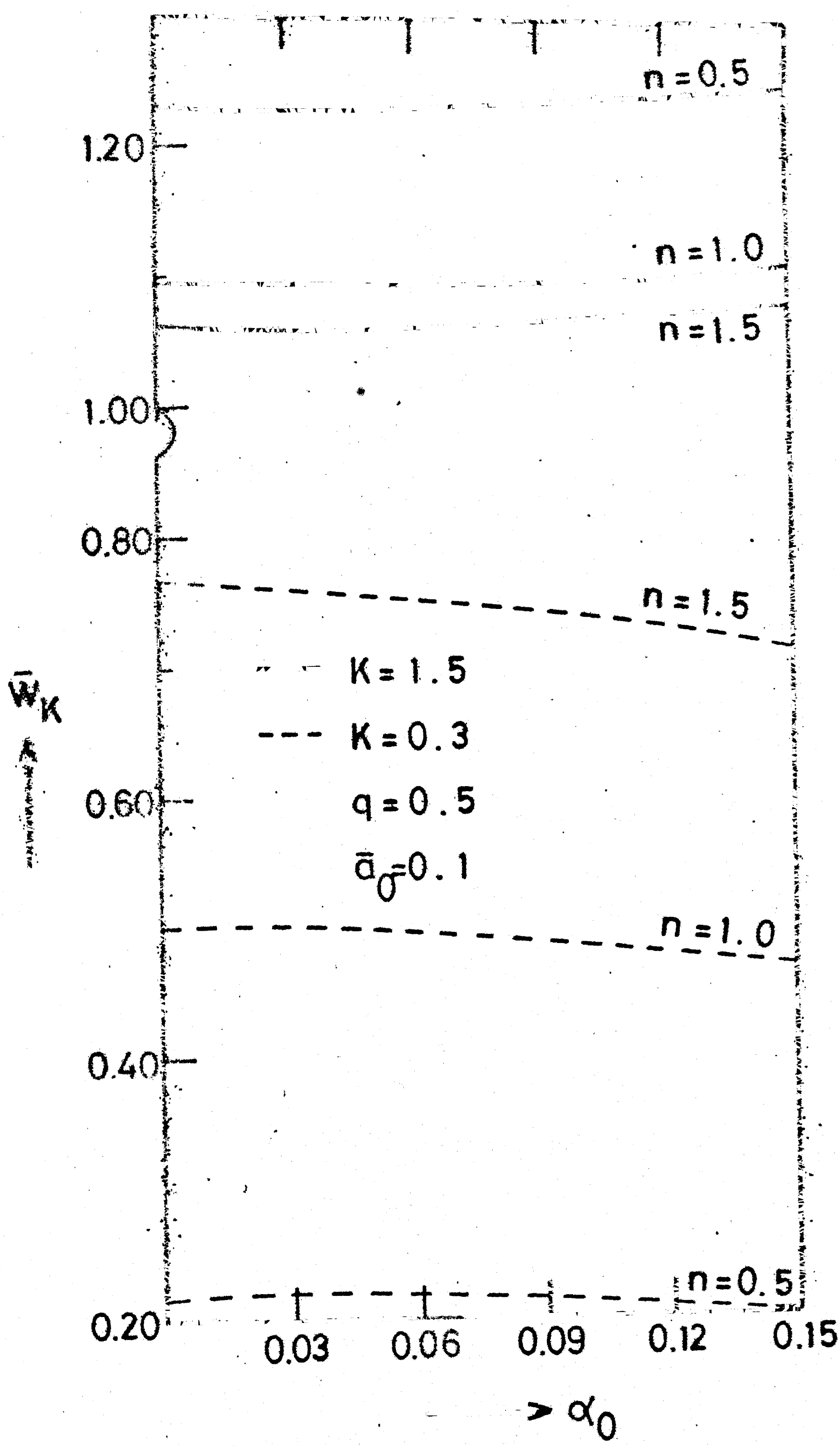

Fig. 3.11 Variation of load ratio $\bar{W}_K$ with the factor α_0 for roller bearing.

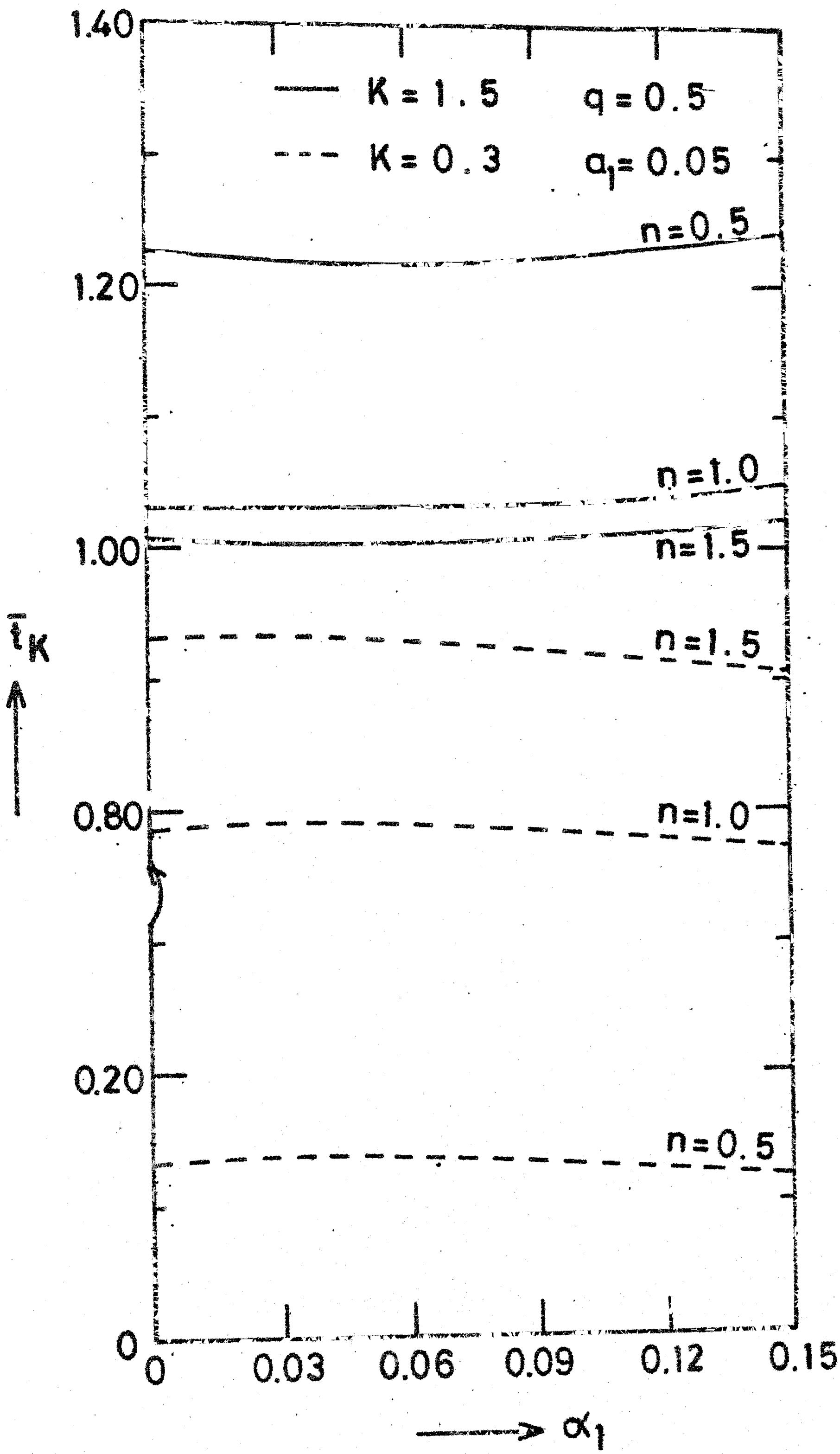

Fig. 3.12 Variation of response time ratio $\bar{t}_K$ with the factor α_1 for roller bearing.

Fig. 3.11 that an increase in peripheral layer thickness increases the load ratio for $K > 1$ and decreases it for $K < 1$. The effect of peripheral layer for $K > 1$ is to increase the load capacity more for pseudoplastics compared to dilatants. The trend is reversed for $K < 1$. Similar interpretations can be made for response time ratio $\bar{t}_K$ vs. α_1 (Fig.3.12).

Calculations of $\bar{W}_q$ and $\bar{t}_q$ from eqns. (3.37) and (3.38) help to study effect of q on these characteristics, which are tablulated against q in Table 1. We observe that the decrease in the load ratio and response time with increasing q is enhanced for pseudoplastics.

Table 1

Variation of $\bar{W}_q$ and $\bar{t}_q$ with q for various values of n

	$\bar{W}_q$			$\bar{t}_q$		
q	n=0.5	n=1.0	n=1.5	n=0.5	n=1.0	n=1.5
0	1.000	1.0000	1.0000	1.0000	1.0000	1.0000
.2	.8839	.8848	.8856	.9479	.9768	.9854
.4	.7813	.7830	.7843	.9001	.9546	.9714
.6	.6907	.6929	.6947	.8563	.9333	.9577

$K = 1.5$, $\bar{a} = 0.1$, $\alpha_o = 0.15$, $\alpha_1 = 0.15$.

3.5 JOURNAL BEARINGS

The problem considered is the flow of a power law lubricant between two eccentric cylinders which approach each other with a relative squeeze velocity V. The velocity V is assumed to be constant both in magnitude and direction and symmetrical with respect to the boundaries of the bearing system. The clearance (difference in the radii) between the cylindrical surfaces is small compared to the radius of the inner cylinder. The configuration of the system is as shown in Fig. 3.13.

With the usual assumptions of lubrication theory, the governing eqns. of motion for a power law lubricant in one dimensional form are given by

$$\frac{dp}{dx} = \frac{\partial}{\partial y}\left\{\left(m \left|\frac{\partial u}{\partial y}\right|^{n-1}\right)\frac{\partial u}{\partial y}\right\} \tag{3.42}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.43}$$

To determine velocity u from eqn. (3.42) appropriate sign is to be attached to the velocity gradient $\frac{\partial u}{\partial y}$. For this purpose let us assume that $\frac{\partial u}{\partial y}$ is zero at a film height $y = \bar{h}(x)$; then, in the region $0 \leq y \leq \bar{h}$, $\frac{\partial u}{\partial y} \geq 0$ and in the region $\bar{h} \leq y \leq h$, $\frac{\partial u}{\partial y} \leq 0$.

Considering the region $\frac{\partial u}{\partial y} \leq 0$, eqn. (3.42) can be written as

$$-\frac{dp}{dx} = \frac{\partial}{\partial y}\left\{m\left(-\frac{\partial u}{\partial y}\right)^n\right\} \tag{3.44}$$

Integrating eqn. (3.44) with the boundary conditions

$$\frac{\partial u}{\partial y} = 0 \quad \text{at } y = \bar{h}$$
$$u = 0 \quad \text{at } y = h \tag{3.45}$$

the velocity profile is obtained as

$$u = \left(-\frac{dp}{dx}\right)^{1/n} \int_y^h \left(\frac{y-\bar{h}}{m}\right)^{1/n} dy \quad \bar{h} \leq y \leq h \tag{3.46}$$

Similarly, considering the region $\frac{\partial u}{\partial y} \geq 0$, we have

$$u = \left(-\frac{dp}{dx}\right)^{1/n} \int_0^y \left(\frac{\bar{h}-y}{m}\right)^{1/n} dy \quad 0 \leq y \leq \bar{h}. \tag{3.47}$$

The velocities are continuous at $y = \bar{h}$, hence from eqns. (3.46) and (3.47) we get

$$\int_{\bar{h}}^{h} \left(\frac{y-\bar{h}}{m}\right)^{1/n} dy = \int_0^{\bar{h}} \left(\frac{\bar{h}-y}{m}\right)^{1/n} dy \tag{3.48}$$

where the consistency m is given by (Fig.3.13)

$$\begin{aligned} m &= Km_1(h/h_1)^q & 0 \leq y < a \\ &= m_1(h/h_1)^q & a < y < h-a \\ &= Km_1(h/h_1)^q & h-a < y \leq h. \end{aligned} \tag{3.49}$$

Using eqn. (3.49) in eqn. (3.48) we get $\bar{h} = h/2$ for $a < \bar{h}$.

The volume flow flux of the lubricant is given by

$$Q = \int_0^h u \, dy \tag{3.50}$$

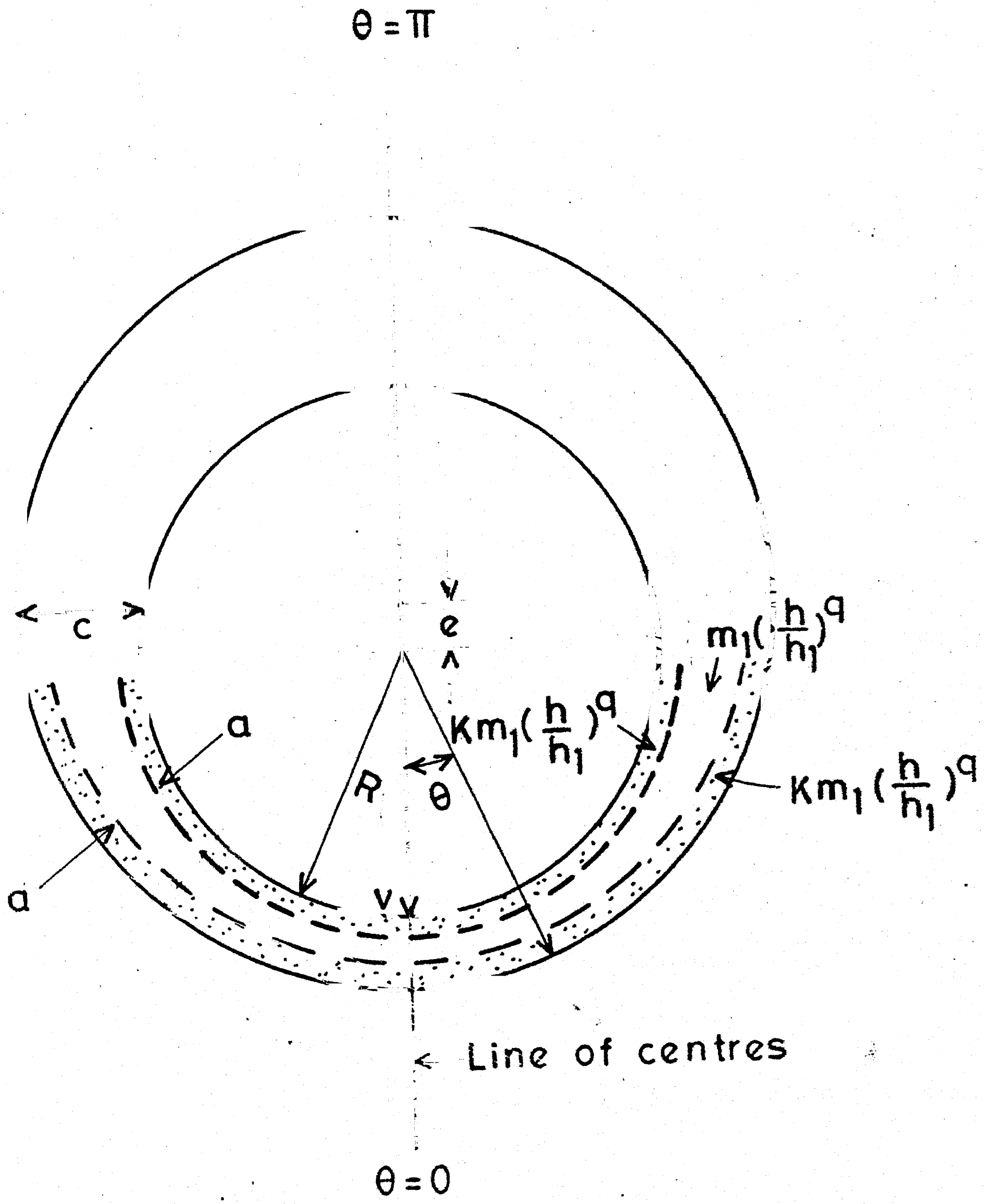

Fig. 3.13 Squeezing in journal bearing.

Using eqns. (3.46), (3.47) and (3.49) in eqn. (3.50) we get

$$Q = \frac{2n}{2n+1} \left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} \, h^{(2n+1-q)/n} \, (g_o) \left(- \frac{1}{m_1} \frac{dp}{dx}\right)^{1/n} \tag{3.51}$$

where

$$(g_o) = 1-(1-K^{-1/n})\Big[1-(1-2a/h)^{(2n+1)/n} - \frac{2n+1}{n+1} \Big\{ \left(1- \frac{2a}{h}\right)-\left(1- \frac{2a}{h}\right)^{(2n+1)/n} \Big\}\Big] \tag{3.52}$$

Integrating the eqn. of continuity (3.43) with the boundary conditions

$$\begin{aligned} v &= V \quad \text{at } y = 0 \\ &= 0 \quad \text{at } y = h \end{aligned} \tag{3.53}$$

we get

$$\frac{\partial Q}{\partial x} = -V \tag{3.54}$$

Using the expression for Q from eqn. (3.51) in eqn. (3.54) we obtain

$$\frac{d}{dx}\left[\frac{2n}{2n+1} \left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} \, h^{(2n+1-q)/n} (g_o) \left(- \frac{1}{m_1} \frac{dp}{dx}\right)^{1/n}\right] = -V \tag{3.55}$$

Substituting $x = R\theta$, where R is the radius of the inner cylinder, we have

$$\frac{1}{R}\ \frac{d}{d\theta}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\ h^{(2n+1-q)/n}(g_o)\left(-\frac{1}{m_1 R}\ \frac{dp}{d\theta}\right)^{1/n}\right]$$

$$= -V \tag{3.56}$$

where

$$h = c(1-\varepsilon\cos\theta)\ ,\ h_1 = h(\theta=\pi) = c(1+\varepsilon)$$
$$V = -c\ \frac{d\varepsilon}{dt}\ \cos\theta \tag{3.57}$$

Integrating eqn. (3.56) and noting that $\frac{dp}{d\theta} = 0$ at $\theta = 0$ we have,

$$\frac{dp}{d\theta} = -m_1 R\ 2^{2n+1}\left(\frac{1}{h_1}\right)^q\left(\frac{2n+1}{2n}\ R\,c\ \frac{d\varepsilon}{dt}\ \frac{1}{(g_o)}\right)^n\ \frac{\sin^n\theta}{h^{2n+1-q}} \tag{3.58}$$

Since the pressure profile is symmetric about $\theta = 0$ or $\theta = \pi$, it is sufficient to consider the region $0 \leq \theta \leq \pi$, where $\frac{dp}{d\theta} \leq 0$, to calculate the bearing characteristics. Integrating eqn. (3.58) with the full journal boundary condition viz. $p(\pi) = 0$, the expression for pressure p can be obtained. Denoting p by $p_{K,q}$ we have,

$$p_{K,q} = m_1 R\, 2^{2n+1}\left(\frac{1}{h_1}\right)^q\left(\frac{2n+1}{2n}\ Rc\ \frac{d\varepsilon}{dt}\right)^n \int_\theta^\pi \frac{\sin^n\theta}{(g_o)^n\ h^{2n+1-q}}\ d\theta$$

$$0 \leq \theta \leq \pi \tag{3.59}$$

The load capacity $W_{K,q}$ is given by

$$W_{K,q} = 2\int_o^\pi p_{K,q}^{(\theta)}\ \cos\theta\ R\ d\theta \tag{3.60}$$

which on using eqn. (3.59) becomes

$$W_{K,q} = B_1\left(\frac{d\varepsilon}{dt}\right)^n\left(\frac{1}{h_1}\right)^q \int_o^\pi \frac{\sin^{n+1}\theta}{(g_o)^n\ h^{2n+1-q}}\ d\theta \tag{3.61}$$

where

$$B_1 = 2m_1R^2 2^{2n+1}\left(\frac{2n+1}{2n}Rc\right)^n \tag{3.62}$$

The Squeezing time $t_{K,q}$ for the surfaces to approach from the initial concentric position $\varepsilon = 0$ to a subsequent position, $\varepsilon = \varepsilon_1$, say, is given by

$$t_{K,q} = \left(\frac{B_1}{W_{K,q}}\right)^{1/n}\left(\frac{1}{h_1}\right)^{q/n}\int_0^{\varepsilon_1}\left[\int_0^{\pi}\frac{\sin^{n+1}\theta}{(g_o)^n h^{2n+1-q}}\,d\theta\right]^{1/n}d\varepsilon \tag{3.63}$$

The effects of consistency ratio K on load and response time can be studied by defining the quantities $\bar{W}_K$ and $\bar{t}_K$ as follows:

$$\bar{W}_K = \frac{W_{K,q}}{W_{1,q}} = \frac{I_{K,q}}{I_{1,q}} \tag{3.64}$$

$$\bar{t}_K = \frac{t_{K,q}}{t_{1,q}} = \frac{\int_0^{\varepsilon_1} I_{K,q}^{1/n}\,d\varepsilon}{\int_0^{\varepsilon_1} I_{1,q}^{1/n}\,d\varepsilon} \tag{3.65}$$

where

$$I_{K,q} = \int_0^{\pi}\frac{\sin^{n+1}\theta}{(G_o)^n(1-\varepsilon\cos\theta)^{2n+1-q}}\,d\theta \tag{3.66}$$

$$(G_o) = 1-(1-K^{-1/n})\Big[1-(1-2\bar{a}/H)^{(2n+1)/n} - \frac{2n+1}{n+1}\{(1-2\bar{a}/H)-(1-2\bar{a}/H)^{(2n+1)/n}\}\Big] \tag{3.67}$$

$$H = h/c = 1-\varepsilon\cos\theta\,,\quad \bar{a} = a/c \tag{3.68}$$

Similarly, the effect of thermal factor q can be studied on load and squeeze time by defining the quantities

$$\bar{W}_q = \frac{W_{K,q}}{W_{K,o}} = \left(\frac{1}{H_1}\right)^q \frac{I_{K,q}}{I_{K,o}} \tag{3.69}$$

$$\bar{t}_q = \frac{t_{K,q}}{t_{K,o}} = \left(\frac{1}{H_1}\right)^{q/n} \frac{\int_0^{\varepsilon_1} I_{K,q}^{1/n}\, d\varepsilon}{\int_0^{\varepsilon_1} I_{K,o}^{1/n}\, d\varepsilon} \tag{3.70}$$

where $I_{K,q}$ is defined in eqn. (3.66) and $H_1 = h_1/c = 1+\varepsilon$

The quantities $\bar{W}_K$, $\bar{t}_K$, $\bar{W}_q$ and $\bar{t}_q$ defined in eqns. (3.64), (3.65), (3.69) and (3.70) are evaluated for various values of n and depicted in Figs. 3.14-3.20. The effect of K is to increase the load capacity for all n and this increase is enhanced for pseudoplastics for $K > 1$ (Fig. 3.14). There is negligible difference in load ratios for Newtonian and dilatant lubricants as $K(\leq 1)$ increases; these ratios are represented by a single curve. A trend similar to that of load ratio is observed in the case of response time ratio vs. consistency ratio (Fig.3.15).

From Fig. 3.16 we observe that for full journal bearings, due to the effect of q, the load capacity decreases for all n and the decrease is enhanced for dilatants whereas the decrease in the response time is enhanced for pseudoplastics as evident from Fig.3.17. In the case of half journal the effect of q on $\bar{W}_q$ and $\bar{t}_q$ is represented in Table 2. It is seen that the decrease in

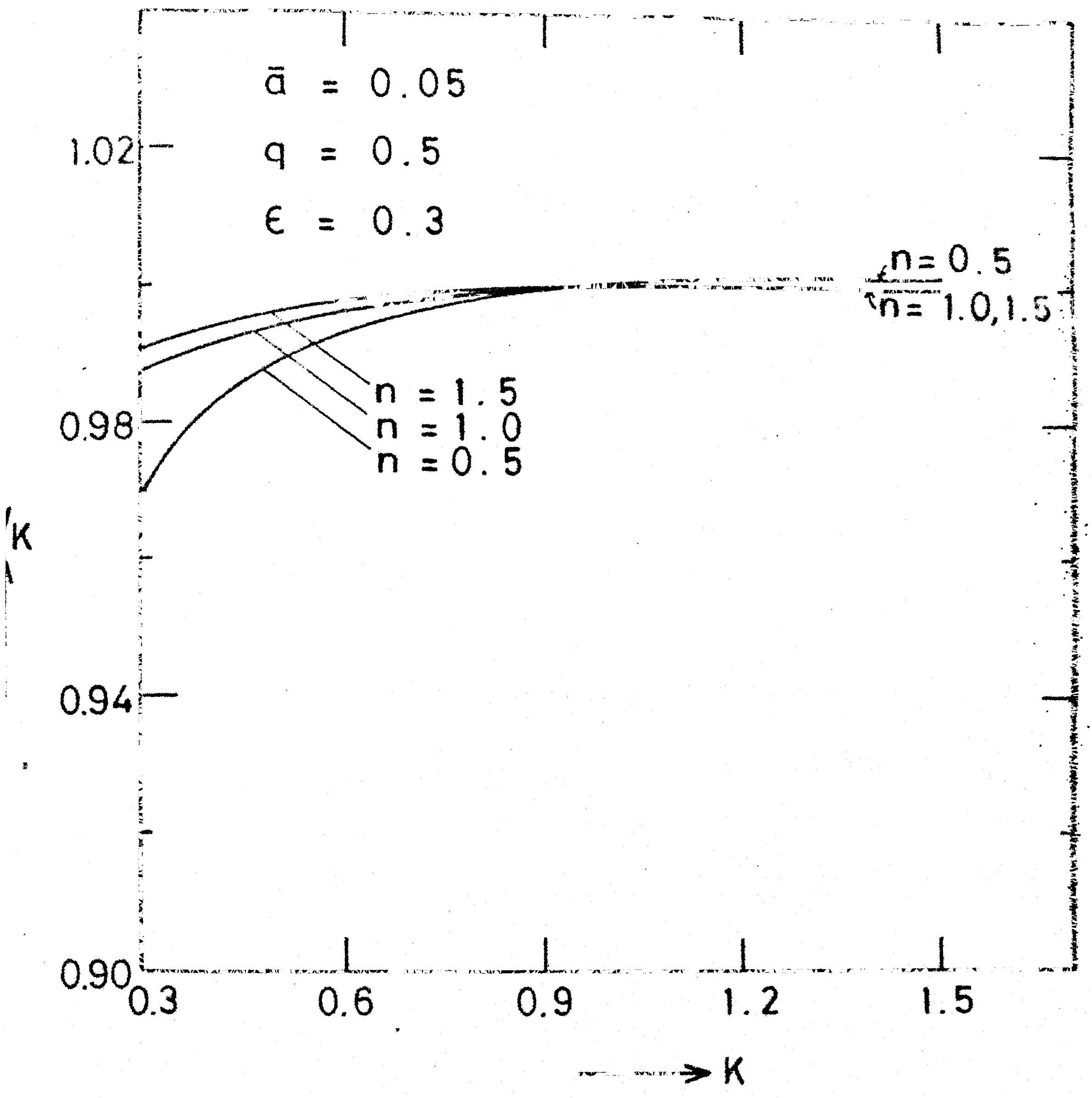

Fig. 3.14 Effect of consistency ratio K on load ratio parameter $\bar{W}_K$ for journal bearing.

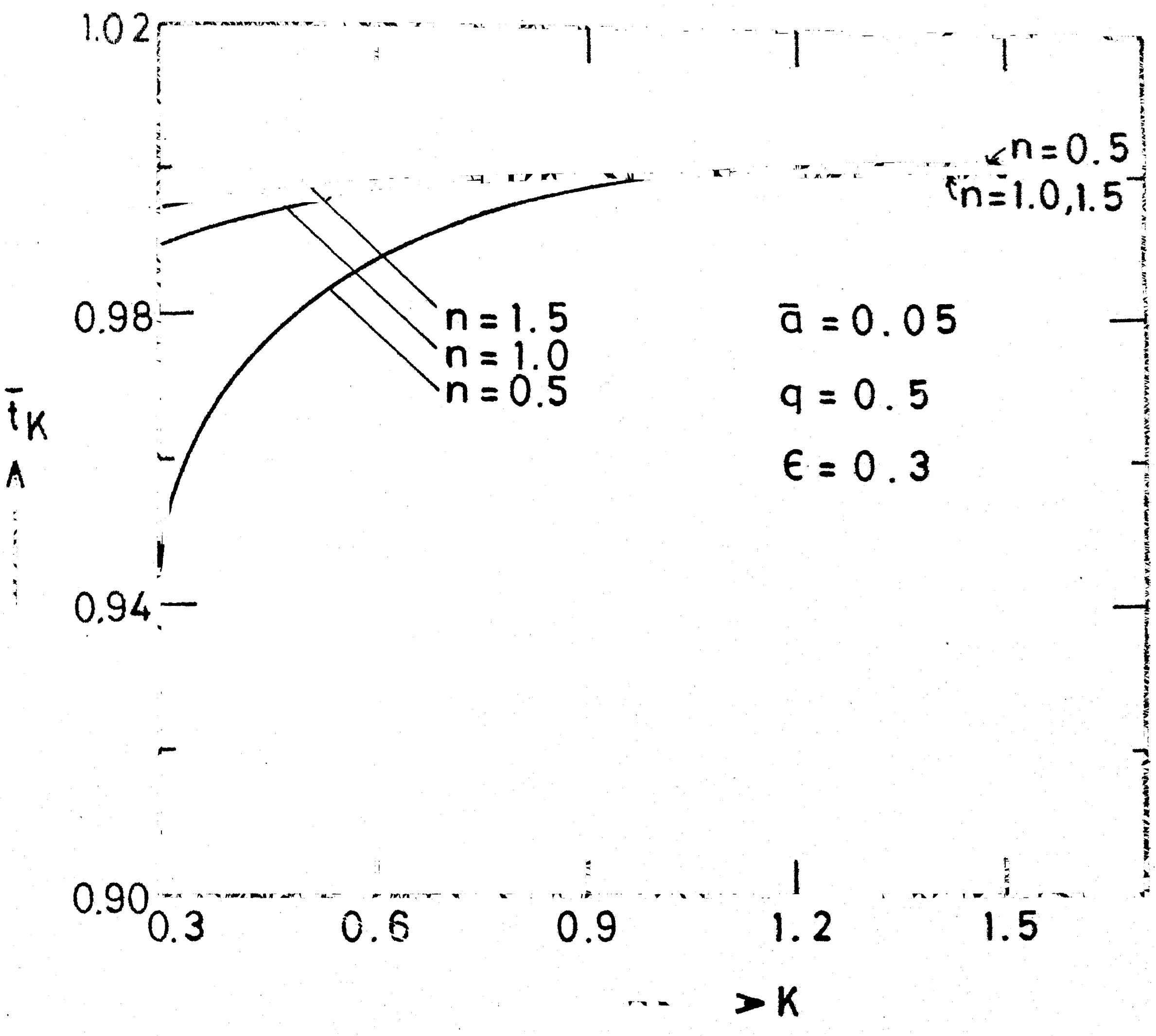

Fig. 3.15 Variation of response time retio $\bar{t}_K$ with consistency ratio K for journal bearing.

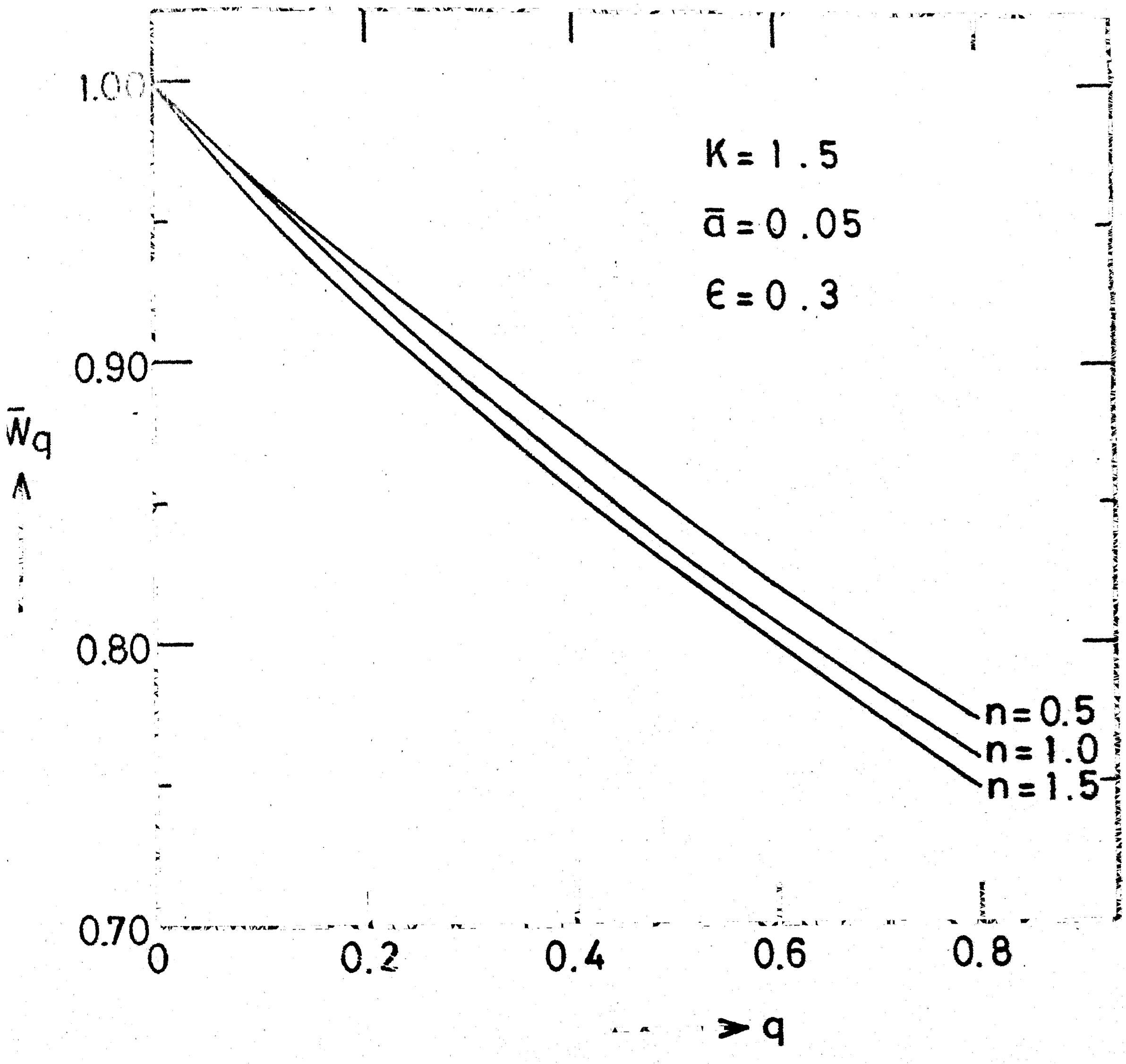

Fig. 3.16 Effect of thermal factor q on load ratio $\bar{W}_q$ for journal bearing.

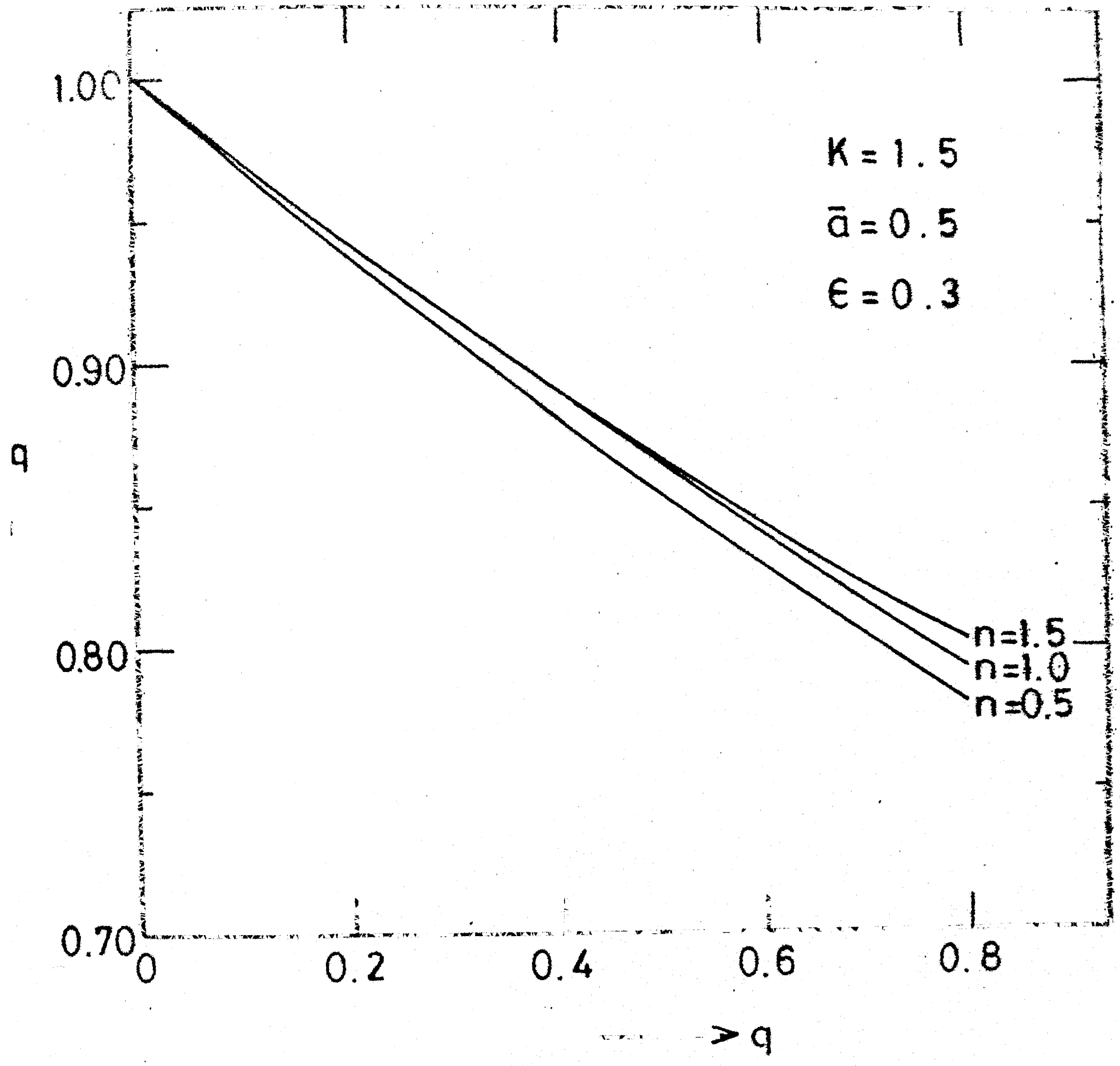

Fig. 3.17 Variation of response time ratio $\bar{t}_q$ with thermal factor q for journal bearing.

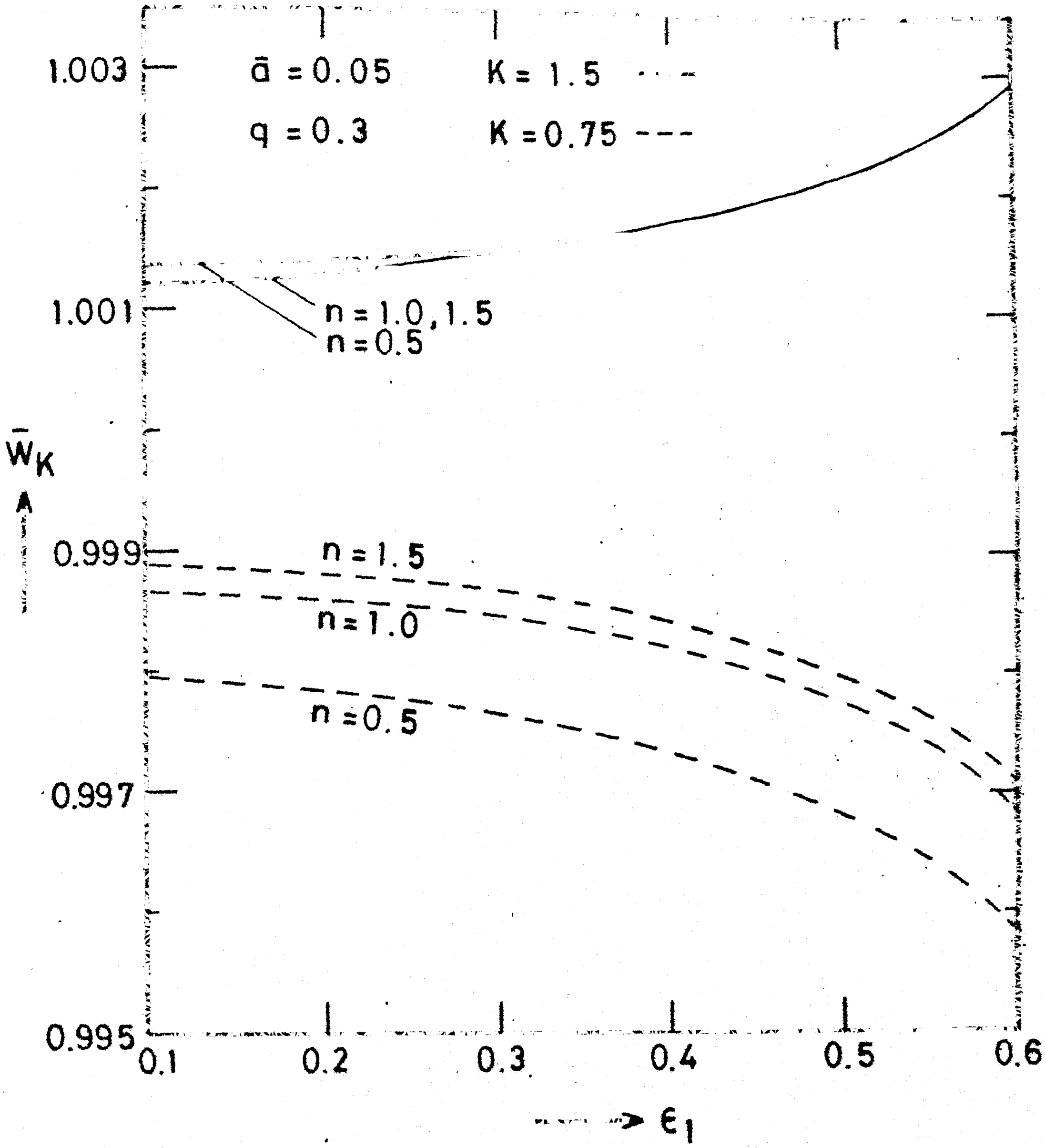

Fig. 3.18 Variation of load ratio $\bar{W}_K$ with final eccentric position ϵ_1 for journal bearing.

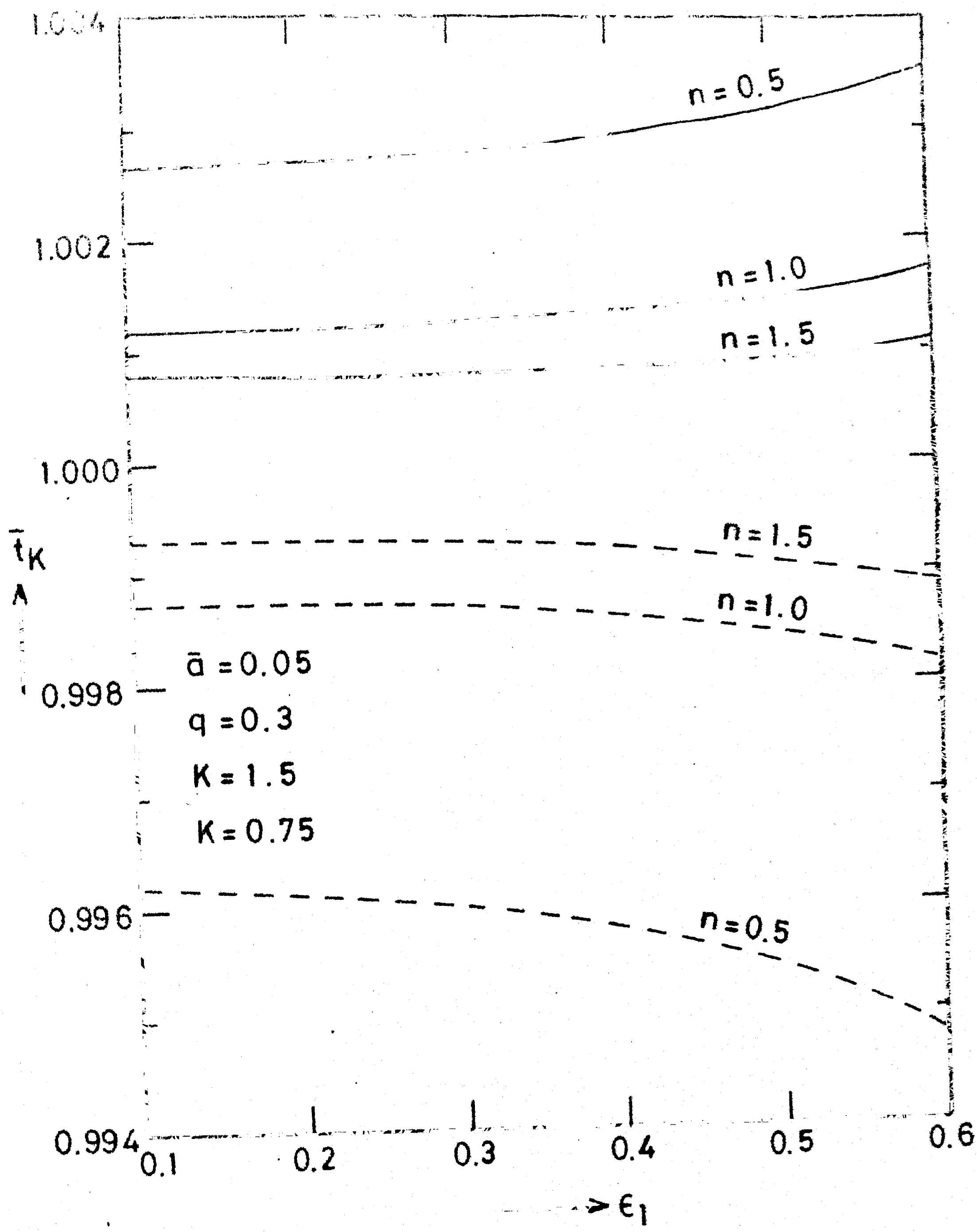

Fig. 3.19 Variation of response time ration $\bar{t}_K$ with final eccentric position ϵ_1 for journal bearing.

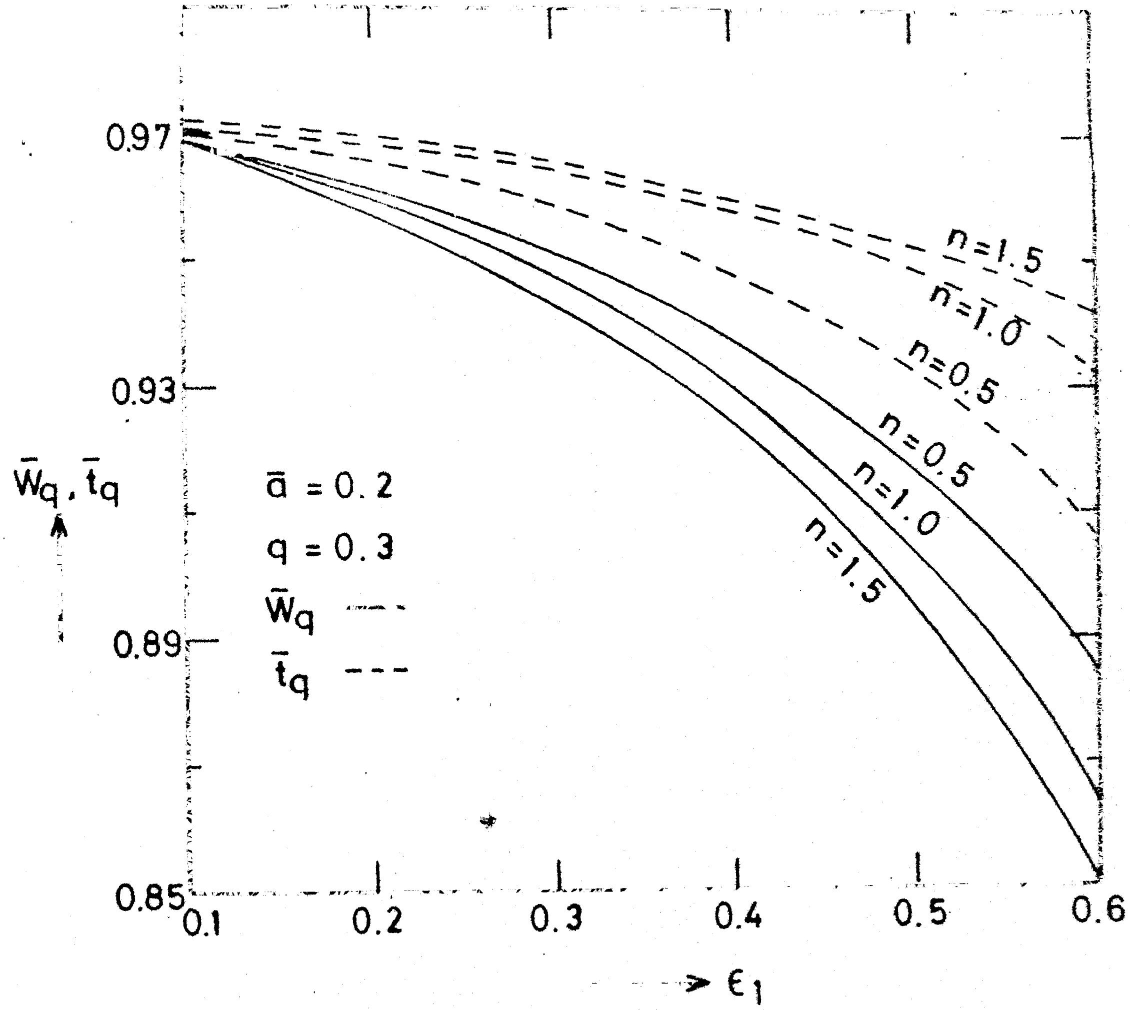

Fig. 3.20 Variation of load ratio $\bar{W}_q$ and response time ratio $\bar{t}_q$ with final eccentric position ϵ_1 for journal bearing.

the load capacity and response time is enhanced for pseudoplastics.

Table 2

Variation of $\bar{W}_q$ and $\bar{t}_q$ with q for half journal bearing for various values of n

	$\bar{W}_q$			$\bar{t}_q$		
q	n = 0.5	n = 1.0	n = 1.5	n = 0.5	n = 1.0	n = 1.5
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
.2	.9169	.9178	.9184	.9653	.9837	.9896
.4	.8411	.8426	.8437	.9325	.9679	.9794
.6	.7718	.7739	.7754	.9013	.9525	.9694

Fig.3.18 represents the variation of $\bar{W}_K$ with ε_1 for full journal bearing. Increase in ε_1 implies a decrease in the film thickness. The load capacity decreases for $K < 1$ and increases for $K > 1$ for all n; however, the difference in the ratio $\bar{W}_q$ disappears for all n as the film thickness sufficiently becomes smaller. For $K < 1$, the decrease in the load capacity is enhanced for pseudoplastic behaviour of the lubricant. Similar trend is observed in Fig. 3.19 for $\bar{t}_K$ vs. ε_1 except that the increase or decrease of response time due to the decrease of film thickness is enhanced for pseudoplastics even for

small values of thickness. As the film thickness decreases, the effect of q enhances the decrease in the load capacity for dilatants and enhances the decrease in the response time for pseudoplastics (Fig.3.20).

3.6 SPHERICAL BEARINGS

Consider squeezing of a power law lubricant between two eccentric spherical surfaces of radii r and R $(r > R)$, approaching each other with a relative squeeze velocity V. The velocity is assumed to be constant in magnitude and in direction and symmetrically placed with respect to the boundaries of the system (Fig.3.21). The film thickness is given by $h = c(1-\varepsilon\cos\theta)$ where $c = r-R$ and $\varepsilon = e/(r-R)$, ε being eccentricity ratio. Following a precedure similar to that adopted in the previous section, one can obtain the velocity u of the lubricant as

$$u = \left(-\frac{dp}{dx}\right)^{1/n} \int_y^h \left(\frac{y-h/2}{m}\right)^{1/n} dy \qquad \frac{\partial u}{\partial y} \leq 0, \; h/2 \leq y \leq h \tag{3.71}$$

$$= \left(-\frac{dp}{dx}\right)^{1/n} \int_0^y \left(\frac{h/2-y}{m}\right)^{1/n} dy \qquad \frac{\partial u}{\partial y} \geq 0, \; 0 \leq y \leq h/2 \tag{3.72}$$

For a sphere the amount of lubricant passing through a conical element of the surface is given by

$$Q = \int_0^h 2\pi R \sin\theta \, u \, dy \tag{3.73}$$

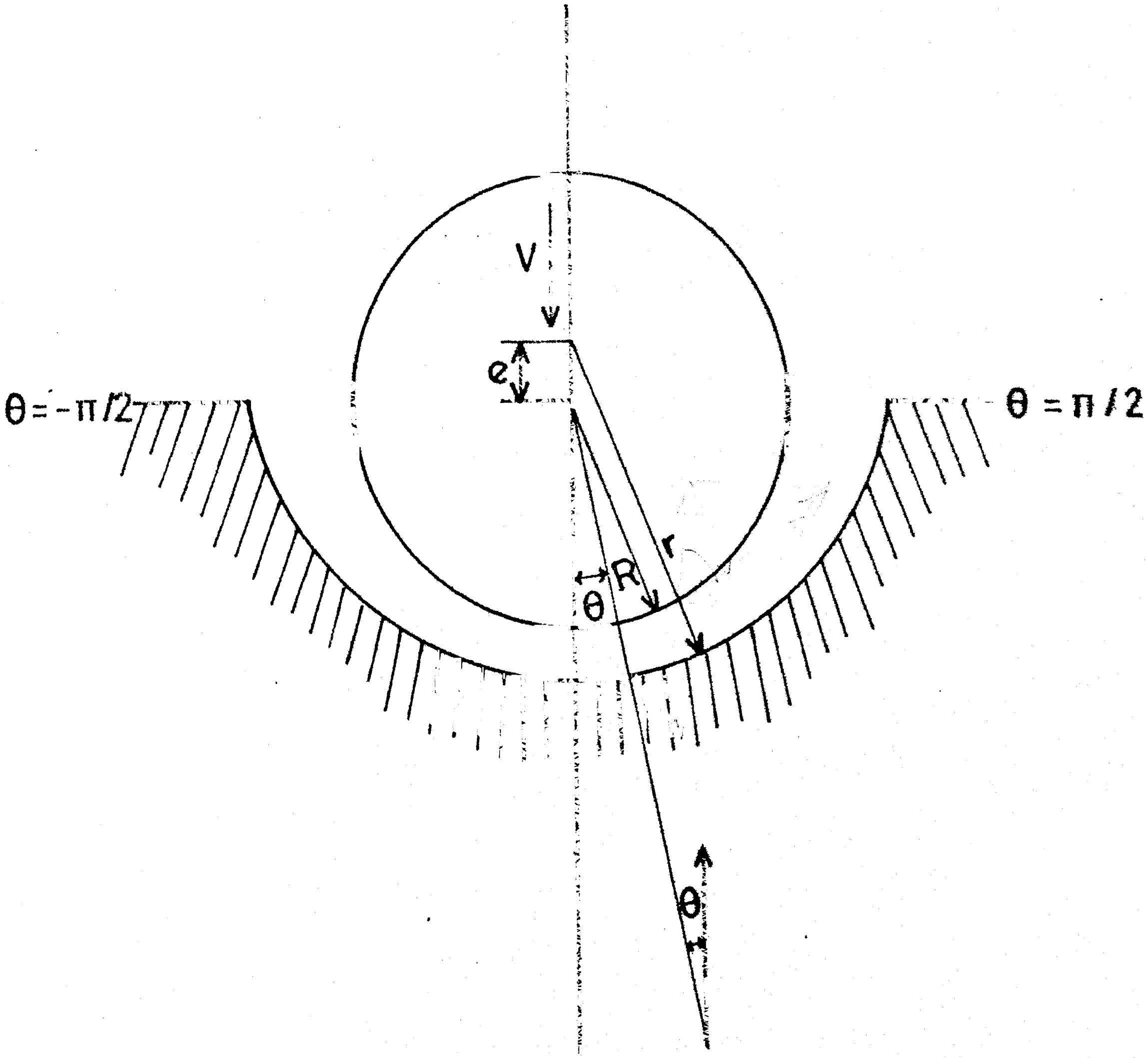

Fig. 3.21 Spherical bearing configuration.

Expressing eqns. (3.71) and (3.72) in polar coordinates and substituting in eqn. (3.73) we get

$$Q = 2\pi R\left(\frac{2n}{2n+1}\right)\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(g_o)\sin\theta\left(-\frac{1}{m_1 R}\frac{dp}{d\theta}\right)^{1/n} \tag{3.74}$$

where

$$(g_o)=1-(1-K^{-1/n})\left[1-\left(1-\frac{2a}{h}\right)^{(2n+1)/n}-\frac{2n+1}{n+1}\left\{\left(1-\frac{2a}{h}\right)-\left(1-\frac{2a}{h}\right)^{(2n+1)/n}\right\}\right] \tag{3.75}$$

h_1 being the inlet film thickness measured at $\theta = \pi/2$

The displacement of lubricant at any point in the seat is given by [17]

$$Q = \pi VR^2 \sin^2\theta \tag{3.76}$$

From eqns. (3.74) and (3.76) we get

$$\frac{dp}{d\theta} = -m_1 R\left(\frac{2n+1}{2n}\,\frac{VR}{2}\right)^n 2^{2n+1}\left(\frac{1}{h_1}\right)^q \cdot \frac{\sin^n\theta}{(g_o)^n h^{2n+1-q}} \tag{3.77}$$

where $V = c\,\frac{d\varepsilon}{dt}$.

Since the pressure profile is symmetric about $\theta = 0$, we shall consider the region $0 \le \theta \le \pi/2$ where $\frac{dp}{d\theta} \le 0$.

Integrating eqn. (3.77) with boundary condition $p(\pi/2) = 0$ we obtain the pressure. Denoting it by $p_{K,q}$ we have

$$p_{K,q}^{(\theta)} = m_1 R\left(\frac{Rc}{2}\,\frac{d\varepsilon}{dt}\,\frac{2n+1}{2n}\right)^n 2^{2n+1}\left(\frac{1}{h_1}\right)^q \int_\theta^{\pi/2} \frac{\sin^n\theta}{(g_o)^n h^{2n+1-q}}\, d\theta$$

$$0 \le \theta \le \pi/2 \tag{3.78}$$

The load capacity $W_{K,q}$ for hemisphere is given by

$$W_{K,q} = 2\pi R^{2} \int_{o}^{\pi/2} p_{K,q}^{(\theta)} \sin\theta \cos\theta \, d\theta \quad ? \tag{3.79}$$

which on using eqn. (3.78) yields

$\frac{d\varepsilon}{dt}$?

$$W_{K,q} = B_2 \left(\frac{1}{h_1}\right)^q \int_{o}^{\pi/2} \frac{\sin^{n+2}\theta}{(g_o)^n h^{2n+1-q}} \, d\theta \tag{3.80}$$

where

$$B_2 = \pi m_1 R^3 \left(\frac{Rc}{2} \frac{2n+1}{2n}\right)^n 2^{2n+1} \tag{3.81}$$

The squeezing time for the surfaces to approach from the initial concentric position $\varepsilon = 0$ to a subsequent position $\varepsilon = \varepsilon_1$, say , is obtained as

$$t_{K,q} = \left(\frac{B_2}{W_{K,q}}\right)^{1/n} \left(\frac{1}{h_1}\right)^{q/n} \int_{o}^{\varepsilon_1} \left[\int_{o}^{\pi/2} \frac{\sin^{n+2}\theta}{(g_o)^n h^{2n+1-q}} \, d\theta \right]^{1/n} d\varepsilon \tag{3.82}$$

As in the case of journal bearings, we may define the following quantities to study the effects of K and q on the squeeze film lubrication of spherical bearings :

$$\bar{W}_K = \frac{W_{K,q}}{W_{1,q}} = \frac{J_{K,q}}{J_{1,q}} \tag{3.83}$$

$$\bar{t}_k = \frac{\int_o^{\varepsilon_1} J_{K,q}^{1/n} \, d\varepsilon}{\int_o^{\varepsilon_1} J_{1,q}^{1/n} \, d\varepsilon} \tag{3.84}$$

and

$$\bar{W}_q = \frac{W_{K,q}}{W_{K,o}} = \frac{J_{K,q}}{J_{K,o}} \quad (3.85)$$

$$\bar{t}_q = \frac{t_{K,q}}{t_{K,o}} = \frac{\int_o^{\varepsilon_1} J_{K,q}^{1/n}\, d\varepsilon}{\int_o^{\varepsilon_1} J_{K,q}^{1/n}\, d\varepsilon} \quad (3.86)$$

where

$$J_{K,q} = \int_o^{\pi/2} \frac{\sin^{n+2}\theta}{(G_o)^n\, H^{2n+1-q}}\, d\theta \quad (3.99)$$

$$(G_o) = 1-(1-K^{-1/n})\left[1-(1-\frac{2\bar{a}}{H})^{(2n+1)/n} - \frac{2n+1}{n+1}\{(1-\frac{2\bar{a}}{H})-(1-\frac{2\bar{a}}{H})^{(2n+1)/n}\}\right] \quad (3.100)$$

$\bar{a} = a/c$, $H = h/c = 1-\varepsilon\cos\theta$,

3.7 CONCLUSIONS

In this Chapter, the characteristics of various squeeze film bearings with power law lubricant has been investigated taking into account of consistency variation.

The effect of high consistency peripheral layer is to increase the load capacity and response time. The increase is more for pseudoplastics compared to dilatants. For the case of low consistency peripheral layer fluid film the achvex properties are reversed. As the peripheral layer thickness of high consistency increases the load and response time increase as has been shown in the case of roller bearing.

The effect of the thermal factor is to decrease the load capacity and response time. The decrease is more pronounced for pseudoplastics compared to dilatants.

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NOMENCLATURE

a	peripheral layer thickness
$\bar{a}$	non-dimensional quantity corresponding to a
c	radial clearance
d	length of parallel plates/film stretch in roller bearings
$2h$	orh film thickness
$2h_1, 2h_2$	initial and subsequent film thicknesses
$2h_o$	minimum film thicknesses
K	consistency ratio
m_1	consistency index
n	flow behaviour index
p	hydrodynamic pressure
q	thermal factor
r	polar coordinate
R	radius of the circular plate, sphere, journal/ equivalent radius of the roller
t	time
$-t_{K,q}$	response time
t_K, t_q	response time ratios
V	normal velocity
$W_{K,q}$	load capacity
$\bar{W}_K, \bar{W}_q$	load ratios
ε	eccentricity
ε_1	final eccentricity position

CHAPTER 4

EFFECTS OF CONSISTENCY VARIATION FOR LIGHTLY LOADED ROLLERS IN COMBINED ROLLING AND NORMAL MOTION

4.1 INTRODUCTION

In Chapter 2, we studied the lubrication of rollers under pure rolling. But in the actual bearing performance, usually there are transient and periodic forces or displacements imposed on the operating lubricant film and the rollers are subjected to both rolling/sliding and normal motion. Sasaki [1] was one of the earliest investigators to study the dynamic behaviour of roller bearings. Dowson et.al [2,3] made a theoretical study followed by an experimental verification of combined rolling/sliding and normal motion of rollers using a Newtonian fluid. In the case of non - Newtonian fluids, most papers which have appeared hitherto deal with either squeezing or rolling/sliding motion [4-9]. However, recently Sinha et.al [10] theoretically analysed the lubrication of rollers in combined rolling and normal motion by characterizing the non-Newtonian behaviour of the operating lubricant by power law model. They did not give attention to variation of consistency in the lubricated region. It was pointed out by Qvale and Wiltshire [11] that lubricants would have a viscosity near the bearing surfaces different from that of the bulk fluid owing to reaction of additives and surfactants with the bearing surfaces. As fluid

film thermal response is instantaneous and the bearing surface response to thermal effects is rather slow with time, thermal equilibrium, perhaps, be improbable with highly transient conditions in combined rolling and normal motion. The principal objective of this Chapter is to study the qualitative behaviour of combined rolling and normal motion under the assumption of thermal equilibrium.

Keeping this in view, in this Chapter, the effects of consistency variation due to aforesaid factors are considered across as well as along the film thickness in the case of lightly loaded rollers with a power law lubricant in combined rolling and normal motion considering cavitation. The consistency variation is accounted through the model suggested in Chapter 2 (eqn. (2.14)).

4.2 BASIC EQUATIONS

Consider the symmetrical flow of a power law lubricant between two identical rollers of radius r moving with a velocity U and a normal velocity V(Fig.4.1). The pressure and velocity profiles are depicted in Fig. 4.2. Taking into account of the consistency variation of the lubricant film, the governing equations to determine pressure in the fluid region are given by eqns. (2.17) and (2.18):

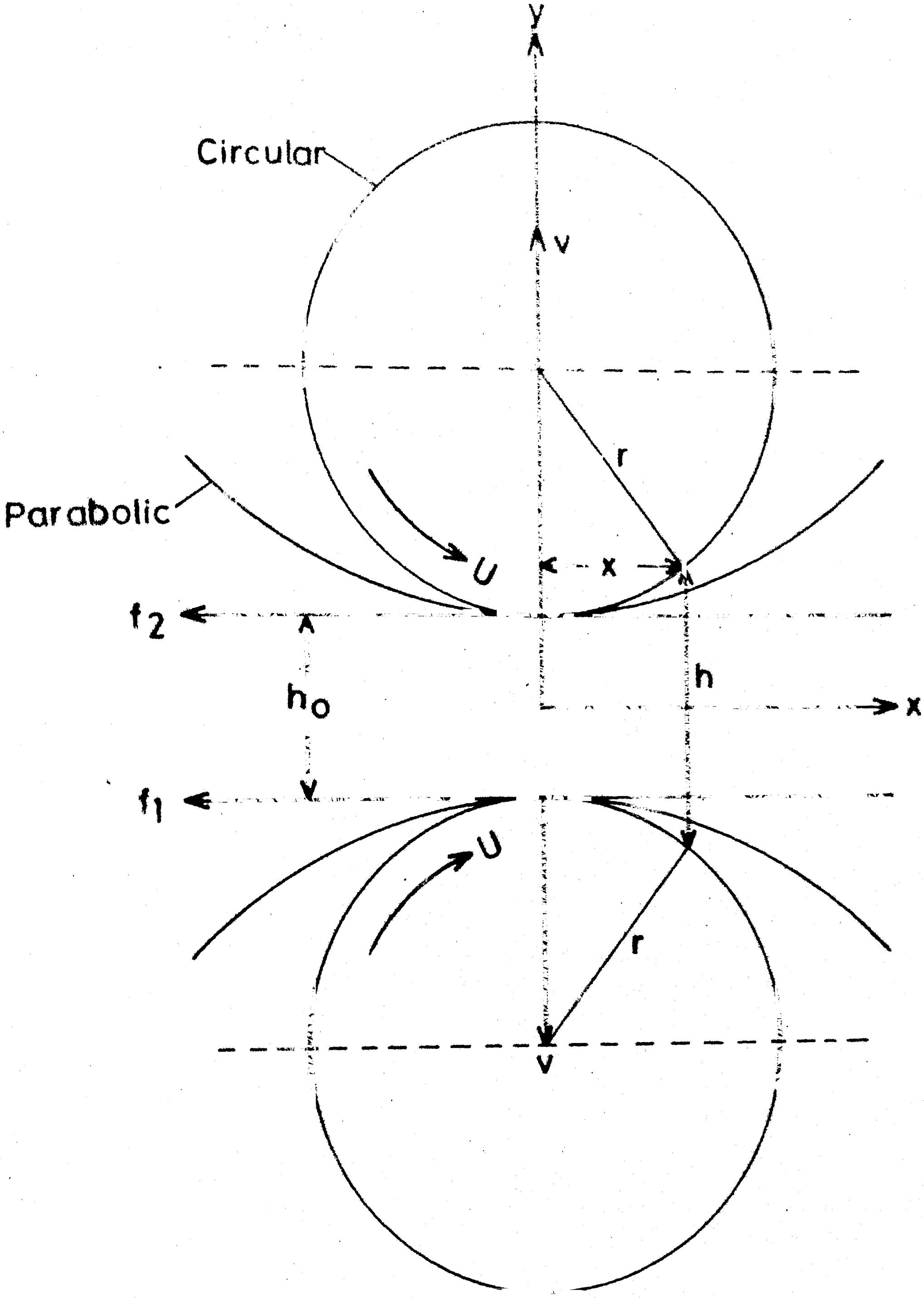

Fig. 4.1 Lubrication of two rollers under rolling and normal motion.

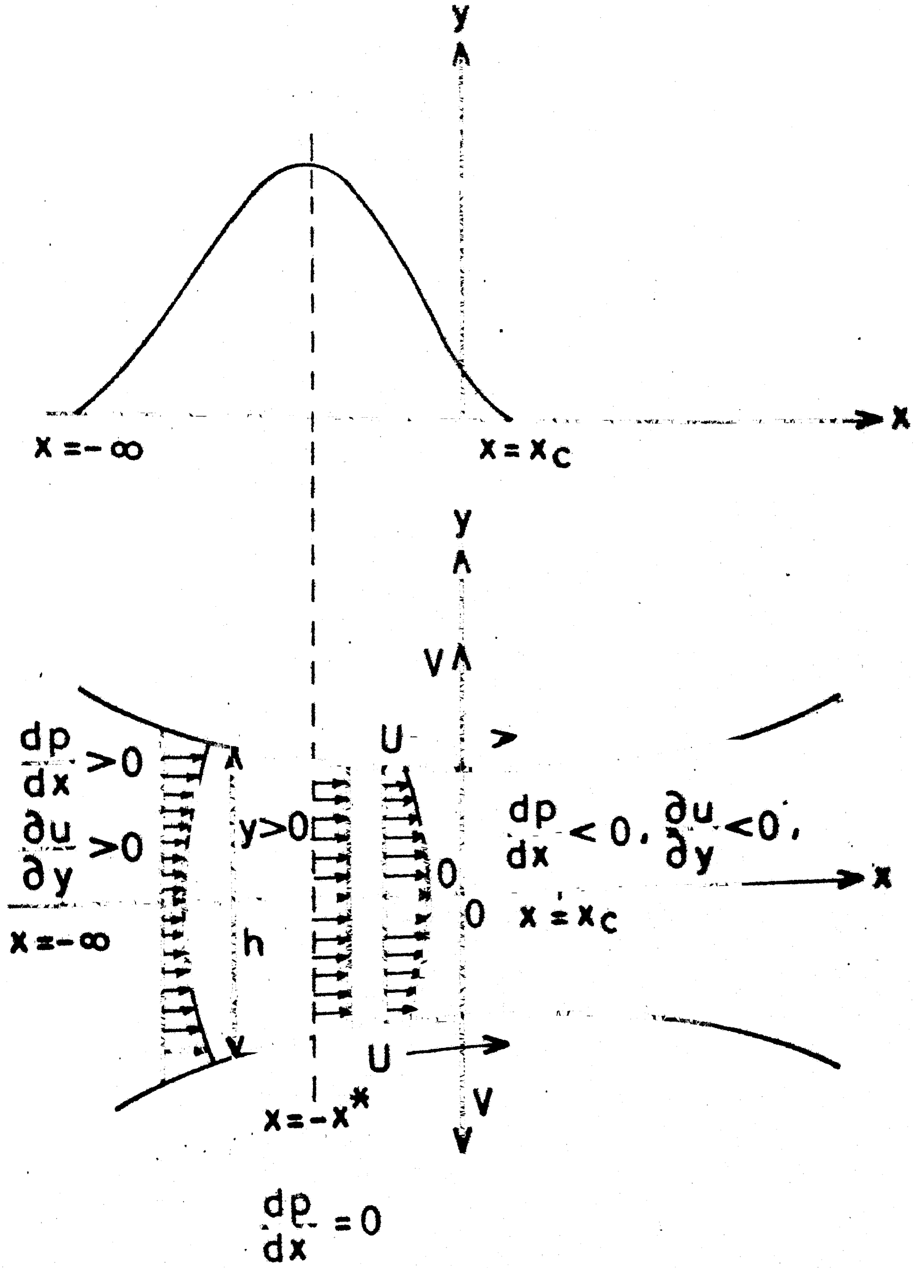

Fig. 4.2 Lubrication of two rollers with power law lubricant.

$$\frac{d}{dx}\left[\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(f_o)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n}\right]$$

$$= \frac{U}{2}\frac{dh}{dx} + V \qquad -\infty \leq x \leq -x^* \qquad (4.1)$$

$$\frac{d}{dx}\left[-\frac{n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\, h^{(2n+1-q)/n}(f_o)\left(-\frac{1}{m_1}\frac{dp_2}{dx}\right)^{1/n}\right]$$

$$= -\left(\frac{U}{2}\frac{dh}{dx} + V\right) \qquad -x^* \leq x \leq x_c \qquad (4.2)$$

where p_1 and p_2 denote the pressure p in the regions defined in eqns. (4.1) and (4.2) respectively, $-x^*$ and x_c are the points of maximum pressure and of cavitation, h_1 is the inlet film thickness at which the thermal factor q is determined and the function (f_o) is given by eqn. (2.16):

$$(f_o) = 1-(1-K^{-1/n})\{1-(1-2a/h)^{(2n+1)/n}\} \qquad (4.3)$$

where K is the consistency ratio and a is the peripheral layer thickness.

The film thickness h is given by

$$h = h_o + x^2/(2R) \qquad (4.4)$$

where h_o is the minimum film thickness and $R = r/2$ is the radius of the equivalent cylinder.

To determine pressure from eqns. (4.1) and (4.2) we use the following boundary conditions (eqns. (2.21)-(2.24):

$$p_1 = 0 \quad \text{at } x = -\infty \qquad (4.5)$$

$$\frac{dp_1}{dx} = \frac{dp_2}{dx} = 0 \text{ at } x = -x^* \tag{4.6}$$

$$p_1 = p_2 \qquad \text{at } x = -x^* \tag{4.7}$$

$$p_2 = \frac{dp_2}{dx} = 0 \qquad \text{at } x = x_c \tag{4.8}$$

Integrating eqns. (4.1) and (4.2) with condition (4.6), we obtain the integrated form of the Reynolds equation as,

$$\frac{dp_1}{dx} = \left(\frac{2n+1}{n}\right)^n 2^{2n+1} m_1 \left(\frac{1}{h_1}\right)^q \frac{\left[\frac{U}{2}(h-h^*)+V(x+x^*)\right]^n}{(f_o)^n\, h^{2n+1-q}}$$

$$-\infty \leq x \leq -x^* \tag{4.9}$$

$$\frac{dp_2}{dx} = -\left(\frac{2n+1}{n}\right)^n 2^{2n+1} m_1 \left(\frac{1}{h_1}\right)^q \frac{\left[\frac{U}{2}(h^*-h)-V(x+x^*)\right]^n}{(f_o)^n\, h^{2n+1-q}}$$

$$-x^* \leq x \leq x_c \tag{4.10}$$

where $h = h^*$ at $x = -x^*$.

Integrating again eqns. (4.9) and (4.10) using conditions (4.5), (4.7) and (4.8), we obtain the expressions determing pressure :

$$p_1(x) = \left(\frac{2n+1}{n}\right)^n 2^{2n+1} m_1 \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{x} \frac{\left[\frac{U}{2}(h-h^*)+V(x+x^*)\right]^n}{(f_o)^n\, h^{2n+1-q}}\, dx$$

$$-\infty \leq x \leq -x^* \tag{4.11}$$

$$p_2(x) = -(\frac{2n+1}{n})^n \, 2^{2n+1} \, m_1 (\frac{1}{h_1})^q \left[\int_{-\infty}^{-x^*} \frac{\{\frac{U}{2}(h-h^*)+V(x+x^*)\}^n}{(f_o)^n \, h^{2n+1-q}} dx \right.$$

$$\left. - \int_{-x^*}^{x} \frac{\{\frac{U}{2}(h^*-h)-V(x+x^*)\}^n}{(f_o)^n \, h^{2n+1-q}} dx \right\} \quad -x^* \leq x \leq x_c \qquad (4.12)$$

Using condition (4.8) in eqn. (4.12) we obtain

$$\int_{-\infty}^{-X^*} I_3(n,K,q,Q)\,dX = \int_{-X^*}^{X_c} I_4(n,K,q,Q)\,dX \qquad (4.13)$$

where

$$I_3(n,K,q,Q) = [(H-H^*)+2Q(X+X^*)]^n/[(F_o)^n H^{2n+1-q}] \qquad (4.14)$$

$$I_4(n,K,q,Q) = [(H^*-H)-2Q(X+X^*)]^n/[(F_o)^n H^{2n+1-q}] \qquad (4.15)$$

$$(F_o) = 1-(1-K^{-1/n})\{1-(1-2\bar{a}/H)^{(2n+1)/n}\} \qquad (4.16)$$

$$X = x/\sqrt{2Rh_o}\,, \quad X^* = x^*/\sqrt{2Rh_o}, \; X_c = x_c/\sqrt{2Rh_o}, \; H = h/h_o = 1+X^2,$$
$$H^* = h^*/h_o = 1+X^{*2}, Q = V\sqrt{(2R/h_o)}/U, \bar{a} = a/h_o \qquad (4.17)$$

A linear relation between X^* and X_c can be obtained from eqns. (4.8), (4.10) and (4.17) as

$$X_c = X^* - 2Q \qquad (4.18)$$

where Q is the dimensionless normal velocity.

The (normal) load W(n,K,q,Q) is given by

$$W(n,K,q,Q) = \int_{-\infty}^{X_c} p\,dx = -\int_{-\infty}^{X_c} x\frac{dp}{dx}\,dx \qquad (4.19)$$

which on using eqns. (4.9) and (4.10) becomes

$$W(n,K,q,Q) = \binom{2n+1}{n}^n 2^{2n+1} m_1 \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-x^*} \frac{x\{\frac{U}{2}(h-h^*)+V(x+x^*)\}^n}{(f_o)^n h^{2n+1-q}} dx - \int_{-x^*}^{x_c} \frac{x\{\frac{U}{2}(h^*-h)-V(x+x^*)\}^n}{(f_o)^n h^{2n+1-q}} dx \right] \qquad (4.20)$$

To study the effect of consistency variation across the film thickness on load, we define the load ratio $\bar{W}_K$ as follows :

$$\bar{W}_K = \frac{W(n,K,q,Q)}{W(n,1,q,Q)} = \frac{\int_{-\infty}^{-X^*} XI_3(n,K,q,Q)dX - \int_{-X^*}^{X_c} XI_4(n,K,q,Q)dX}{\int_{-\infty}^{-X^*_{K1}} XI_3(n,1,q,Q)dX - \int_{-X^*_{K1}}^{X_{cK1}} XI_4(n,1,q,Q)dX} \qquad (4.21)$$

The case K = 1 yields the results corresponding to no consistency variation across the film thickness- that is, the results pertaining to consistency variation along the film thickness only for specified values of n, q, Q and for the same instantaneous minimum film thickness as that of the case with the specified consistency ratio. The points $-X^*_{K1}$ and X^*_{cK1} correspond to points of maximum pressure and cavitation respectively, for the case K = 1.

The effect of consistency variation along the film thickness on load can be studied by defining the quantities $\bar{W}_q$ as,

$$\bar{W}_q = \frac{W(n,K,q,Q)}{W(n,K,0,Q)} = \frac{1}{H_1^q} \frac{\int_{-\infty}^{-X^*} XI_3(n,K,q,Q)dX - \int_{-X^*}^{X_c} XI_4(n,K,q,Q)dX}{\int_{-\infty}^{-X^*_{qo}} XI_3(n,K,0,Q)dX - \int_{-X^*_{qo}}^{X_{cqo}} XI_4(n,K,0,Q)dX} \qquad (4.22)$$

where $-X^*_{qo}$ and X^*_{cqo} are points of maximum pressure and cavitation in the case of q = 0 with the specified values of n, K, Q and with the same instantaneous minimum film thickness and $H_1 = h/h_o$.

Similarly, the effect of squeeze velocity Q on load is studied by defining the quantities $\bar{W}_Q$ as

$$\bar{W}_Q = \frac{W(n,K,q,Q)}{W(n,K,q,O)}$$

$$= \frac{\int_{-\infty}^{-X^*} XI_3(n,K,q,Q)\,dX - \int_{-X^*}^{X_c} XI_4(n,K,q,Q)\,dX}{\int_{-\infty}^{-X^*_{Qo}} XI_3(n,K,q,O)\,dX - \int_{-X^*_{Qo}}^{X_{cQo}} XI_4(n,K,q,O)\,dX} \tag{4.23}$$

where $-X^*_{Qo}$ and X_{cQo} are points of maximum pressure and cavitation in the case of pure rolling (Q=0) with the same instantaneous minimum film thickness.

The frictional force $f_2(n,K,q,Q)$ on the surface $y = h/2$ is given by

$$f_2(n,K,q,Q) = \int_{-\infty}^{X_c} \tau\Big|_{y=h/2}\,dx \tag{4.24}$$

where

$$\tau = m \left|\frac{\partial u}{\partial y}\right|^{n-1} \frac{\partial u}{\partial y} \tag{4.25}$$

Using the expressions for velocity u in different regions obtained in eqns. (2.5) and (2.6) we get,

$$f_2 = -\left[\int_{-\infty}^{-x^*} \frac{h}{2}\frac{dp_1}{dx}\,dx + \int_{-x^*}^{x_c} \frac{h}{2}\frac{dp_2}{dx}\,dx\right] \tag{4.26}$$

substituting the expressions for $\frac{dp_1}{dx}$ and $\frac{dp_2}{dx}$ from eqns.(4.9) and (4.10) in eqn. (4.26) we study the effects of consistency variation across and along the film thickness and normal velocity Q on frictional forces by defining the quantities $\bar{F}_{2K}$, $\bar{F}_{2q}$ and $\bar{F}_{2Q}$, respectively as follows :

$$\bar{F}_{2K} = \frac{f_2(n,K,q,Q)}{f_2(n,1,q,Q)} = \frac{\int_{-\infty}^{-X^*} X^2 I_3(n,K,q,Q)\,dX - \int_{-X^*}^{X_c} X^2 I_4(n,K,q,Q)\,dX}{\int_{-\infty}^{-X^*_{Ko}} X^2 I_3(n,1,q,Q)\,dX - \int_{-X^*_{Ko}}^{X_{cKo}} X^2 I_4(n,1,q,Q)\,dX} \tag{4.27}$$

$$\bar{F}_{2q} = \frac{f_2(n,K,q,Q)}{f_2(n,K,0,Q)} = \frac{1}{H_1^q}\,\frac{\int_{-\infty}^{-X^*} X^2 I_3(n,K,q,Q)\,dX - \int_{-X^*}^{X_c} X^2 I_4(n,K,q,Q)\,dX}{\int_{-\infty}^{-X^*_{qo}} X^2 I_3(n,K,0,Q)\,dX - \int_{-X^*_{qo}}^{X_{cqo}} X^2 I_4(n,K,0,Q)\,dX} \tag{4.28}$$

$$\bar{F}_{2Q} = \frac{f_2(n,K,q,Q)}{f_2(n,K,q,0)} = \frac{\int_{-\infty}^{-X^*} X^2 I_3(n,K,q,Q)\,dX - \int_{-X^*}^{X_c} X^2 I_4(n,K,q,Q)\,dX}{\int_{-\infty}^{-X^*_{Qo}} X^2 I_3(n,K,q,0)\,dX - \int_{-X^*_{Qo}}^{X_{cQo}} X^2 I_4(n,K,q,0)\,dX} \tag{4.29}$$

where the expressions $I_3(n,K,q,Q)$ and $I_4(n,K,q,Q)$ are defined in eqns. (4.14) and (4.15) respectively.

4.3 RESULTS AND DISCUSSIONS

Eqn. (4.13) is numerically evaluated to determine the location of cavitation point for various values of Q and n. Positive values of Q represent the normal separation of rollers and negative values denote the squeeze velocities. However, in this study, the squeeze velocity alone is considered as is the case in most normal bearing performances.

Figs. 4.3 depicts the effect of squeeze velocity on the location of cavitation point for various values of n. As the squeeze velocity increases, the cavituation point moves away from the minimum film thickness. The rate of shifting of cavitation point with respect to squeeze velocity is almost the same for all n except that for pseudoplastics it lies further away from the converging gap than that for dilatants.

In Fig. 4.4-4.11 are studied the variation of load ratio and frictional drag ratio parameters for different n. As in the case of pure rolling, we observe that for specified squeeze velocity, an increase in $\bar{a}$ results in an increase in the load capacity for the case of high consistency peripheral layer fluid film (Fig. 4.4). A similar interpretation can be given to frictional drag ratio parameter $\bar{F}_{2\bar{a}}$ (Fig.4.5).

In Fig. 4.6 is studied the variation of $\bar{W}_K$ with K for various values of n. Again we note that the effective increase in the high consistency peripheral layer is to increase the load capacity for all n. The increase due to the effect of this peripheral layer is to give more load. for dilatants ; however, in the case of low consistency peripheral layer more load is pronounced for pseudoplastics. Similar behaviour is observed for frictional drag ratio parameter (Fig. 4.7).

In Fig. 4.8 is plotted the load ratio $\bar{W}_Q$ against squeeze velocity Q for different values of n. The effect of the squeeze velocity is to increase the pressure generation in the lubrication regime. Due to this effect the load capacity increases for all n. Also, it is reconfirmed that in the case of combined rolling and squeezing the load is more compared to pure rolling. The increase in the load capacity due to squeeze velocity is more pronounced for large values of squeeze velocity. The increase in the load capacity is more for dilatants. A similar interpretations can be given to frictional ratio parameter $\bar{F}_{2Q}$ vs. Q(Fig.4.9).

As in the case of pure rolling, we observe that the effect of the thermal factor q in combined rolling and squeezing is to reduce the load capacity and frictional forces (Figs. 4.10 and 4.11). The reduction is more pronounced for dilatants compared to pseudoplastic behaviour of the lubricant.

4.4 CONCLUSIONS

The effect of the squeeze velocity is to increase the load capacity and frictional drag for all values of the flow behaviour index n. This is because an increase in squeeze velocity means more pressure generation in the lubricated region. The increase is more for dilatants than that for pseudoplastics, as in the case of pure rolling. As the squeeze velocity increases, the cavitation point shifts away from the minimum film thickness.

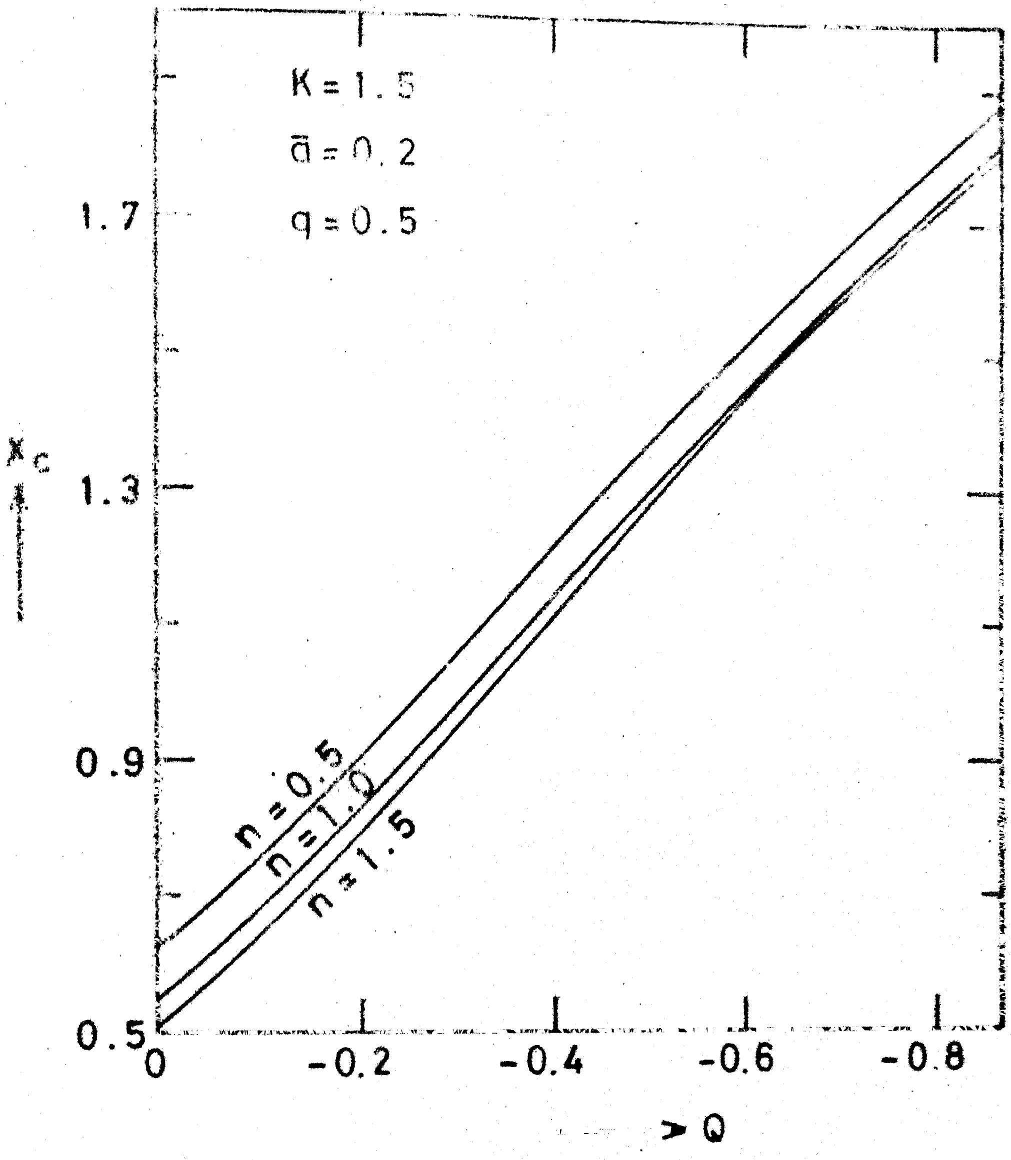

Fig. 4.3 Cavitation point X_c vs. squeeze velocity Q for various values of flow behaviour index n.

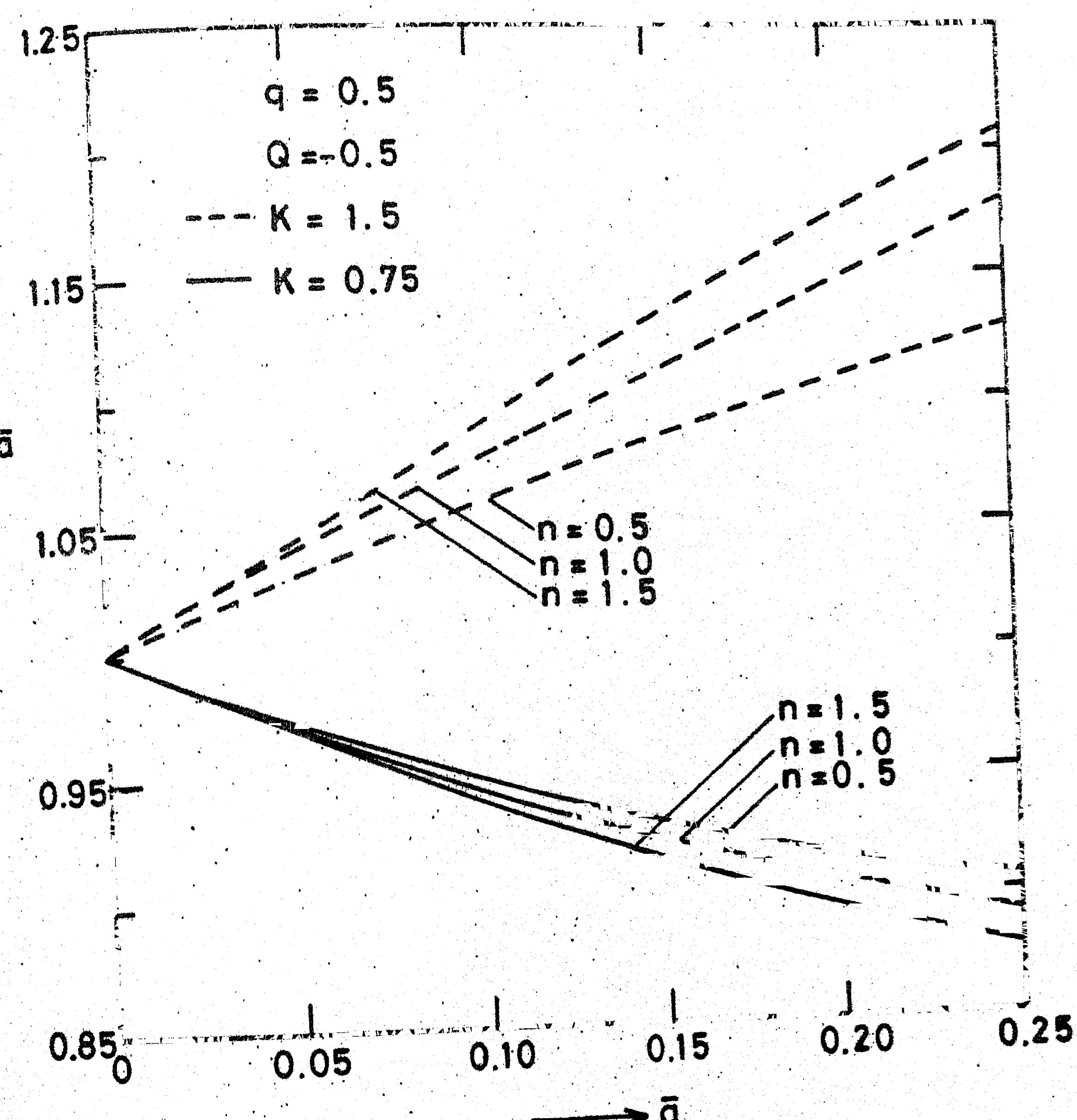

Fig. 4.4 Load ratio parameter $\bar{W}_{\bar{a}}$ vs. peripheral layer thickness $\bar{a}$ for various values of flow behaviour index n, K<1 and K>1.

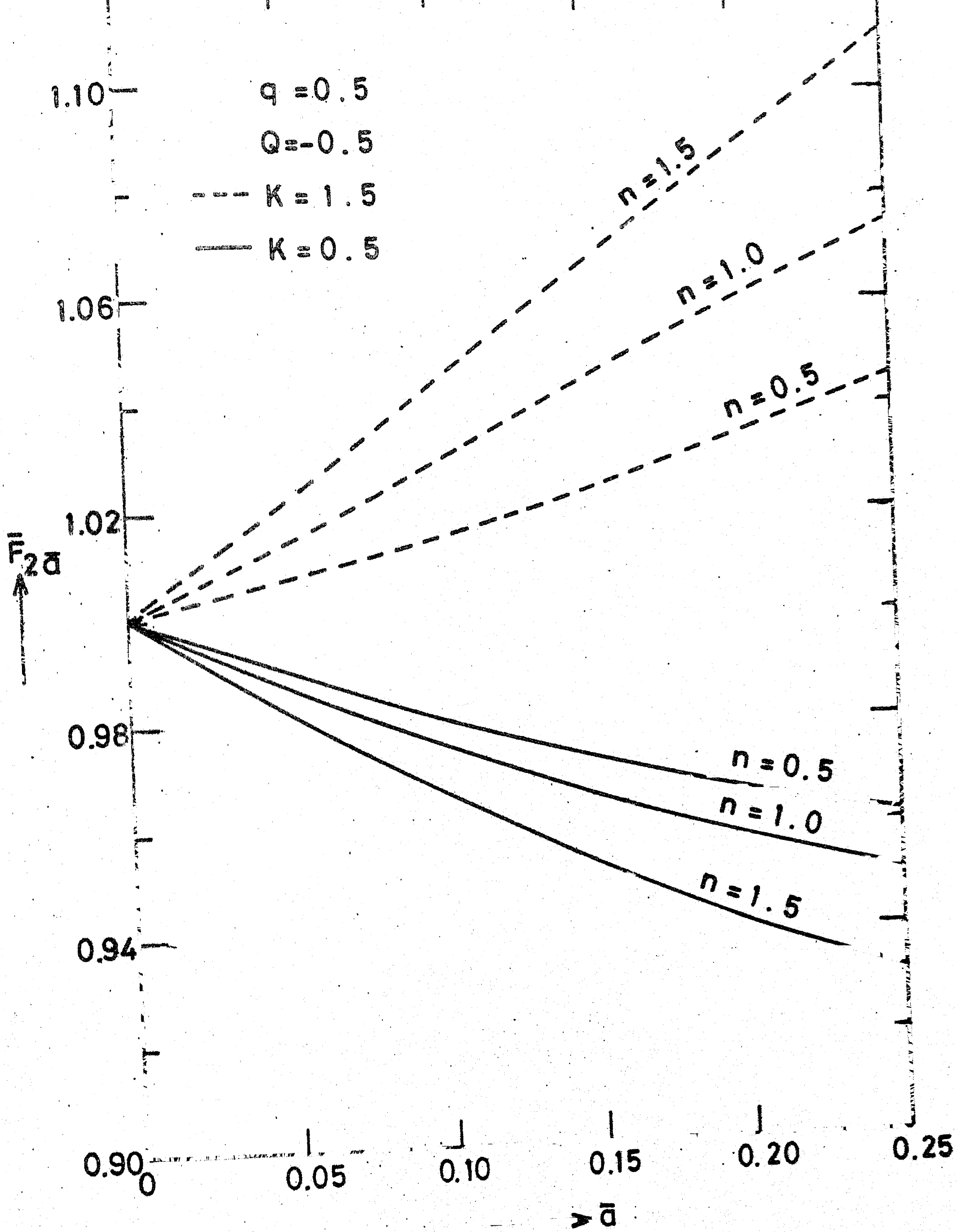

Fig. 4.5 Frictional drag ratio parameter $\bar{F}_{2\bar{a}}$ vs. peripheral layer thickness for various values of flow behaviour index n, K< 1 and K> 1.

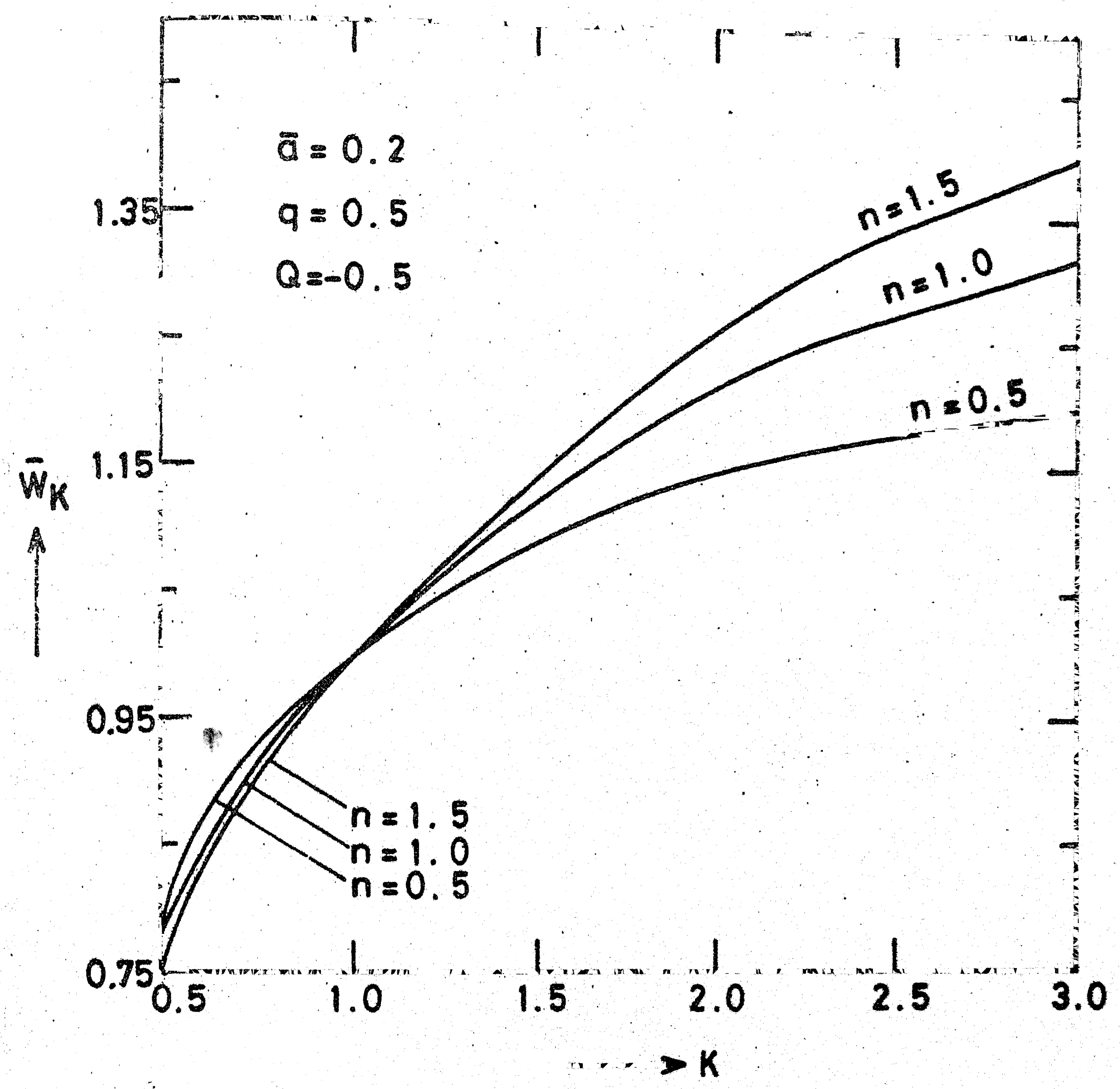

Fig. 4.6 Load ratio parameter $\bar{W}_K$ vs. consistency ratio K for various values of flow behaviour index n.

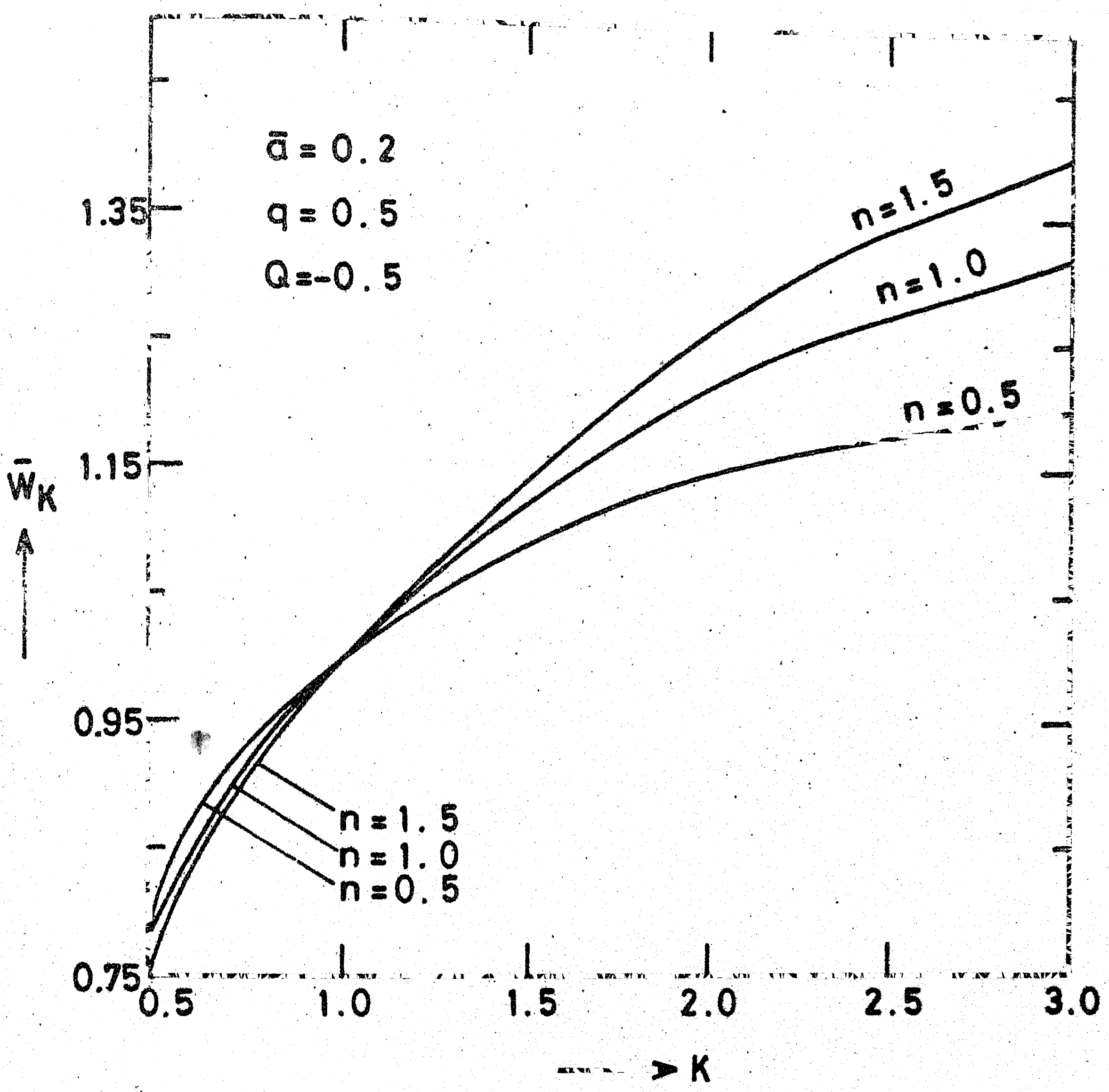

Fig. 4.6 Load ratio parameter $\bar{W}_K$ vs. consistency ratio K for various values of flow behaviour index n.

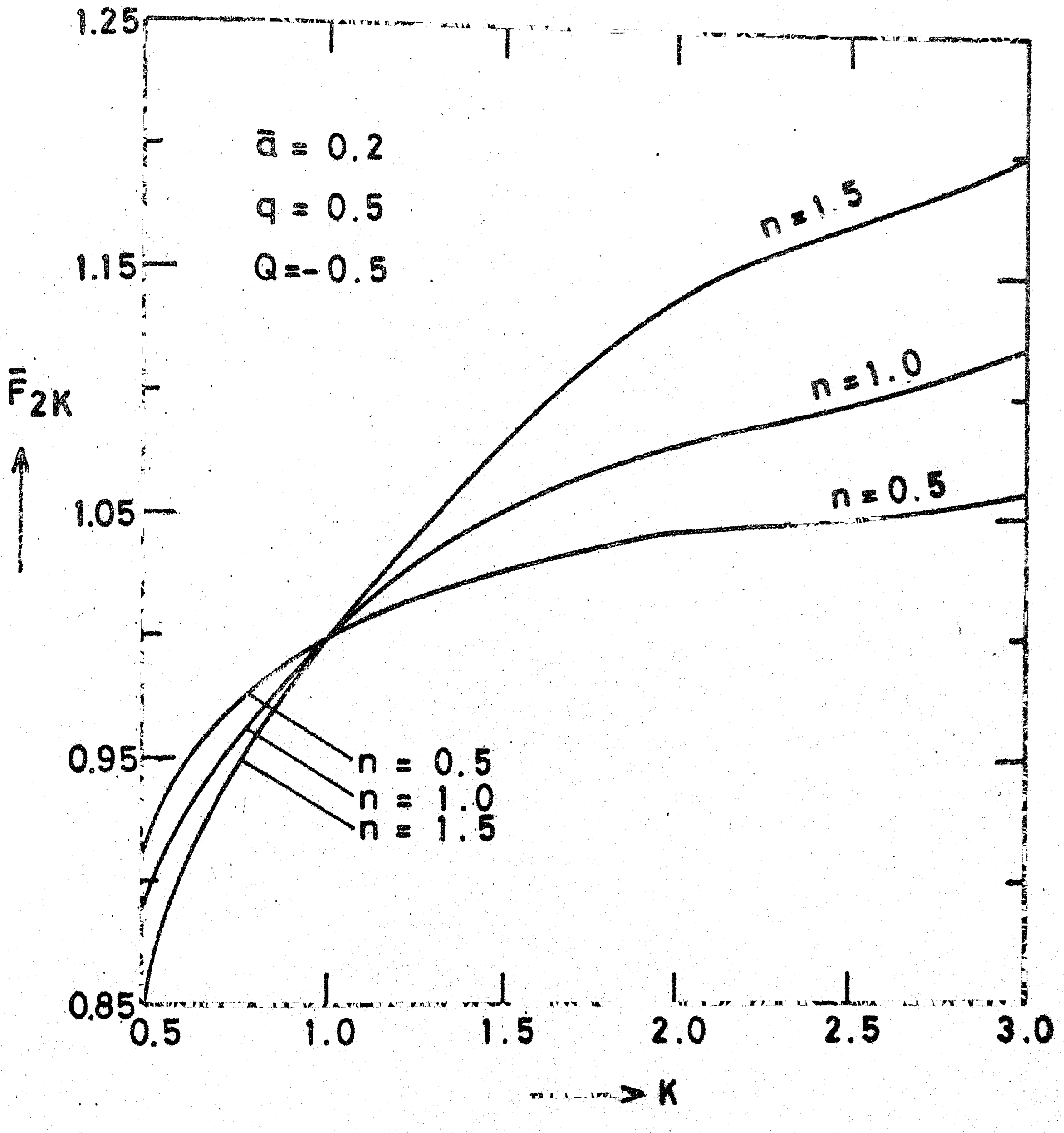

Fig. 4.7. Frictional drag ratio parameter $\bar{F}_{2K}$ vs. consistency ratio K for various values of flow behaviour index n.

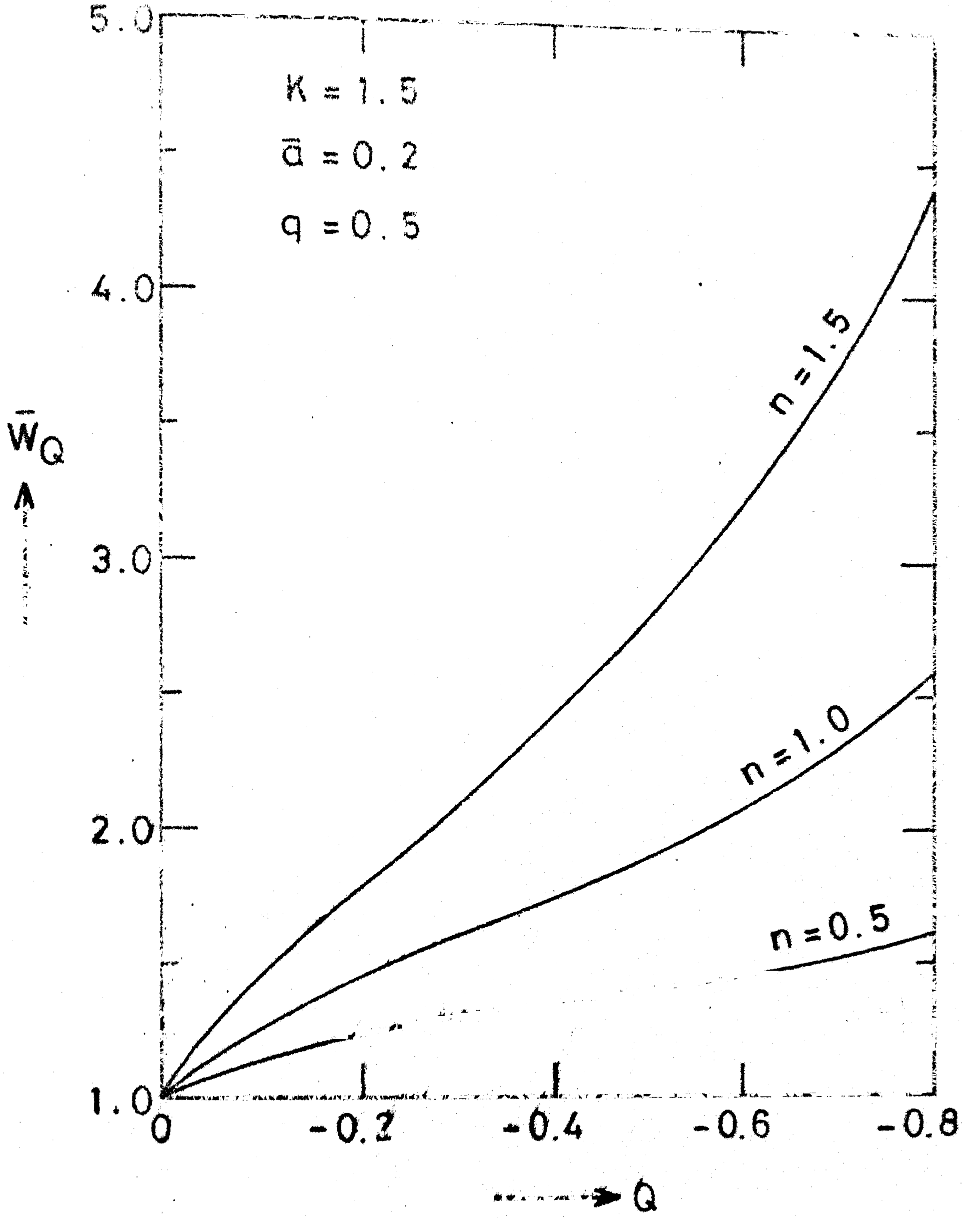

Fig. 4.8 Load ratio parameter $\bar{W}_Q$ vs. squeeze velocity Q for various values of flow behaviour index n.

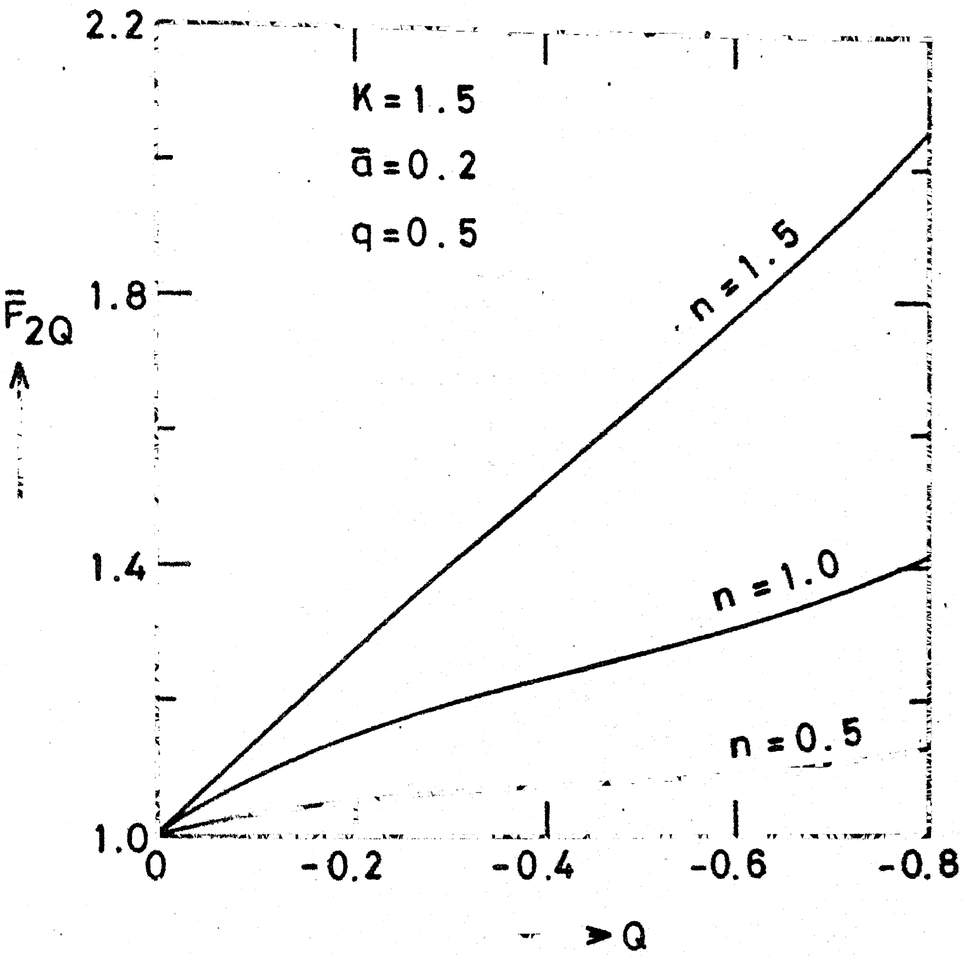

Fig. 4.9 Frictional drag ratio parameter $\bar{F}_{2Q}$ vs. squeeze velocity for various values of flow behaviour index n.

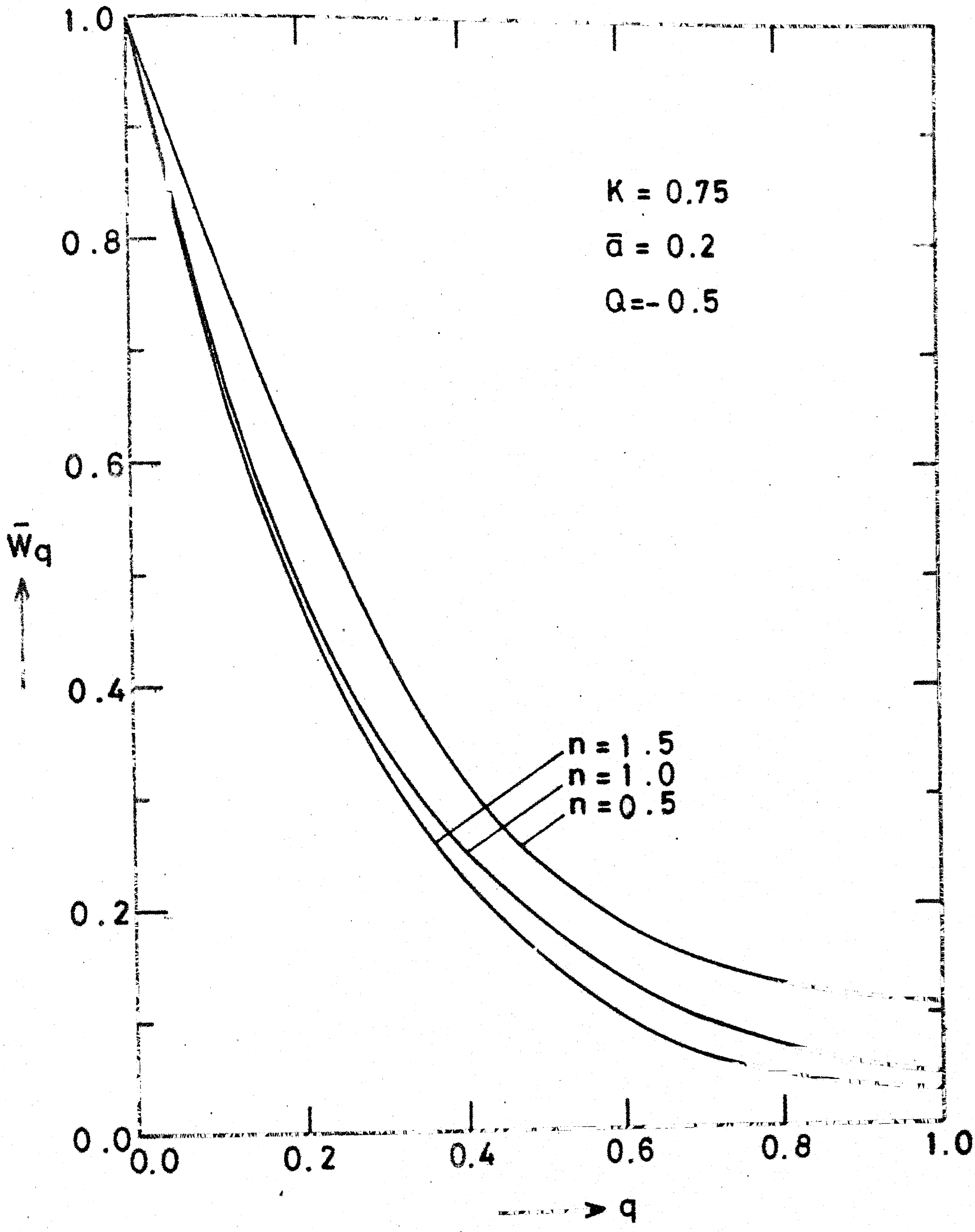

Fig. 4.10 Load ratio parameter $\bar{W}_q$ vs. thermal factor q for various values of flow behaviour index n.

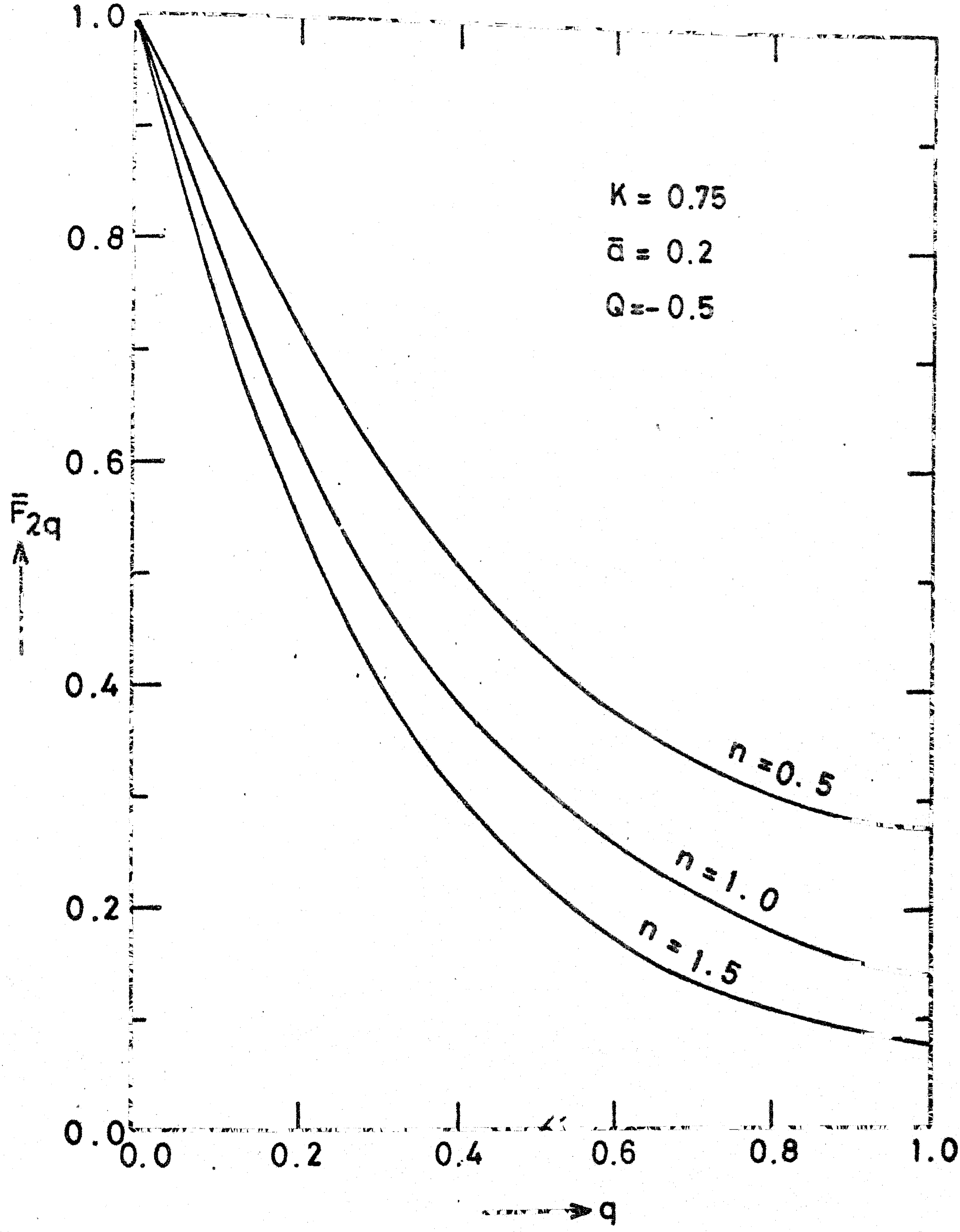

Fig. 4.11 Frictiona drag ratio parameter $\bar{F}_{2q}$ vs. thermal factor q for various values of flow behaviour index n .

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NOMENCLATURE

a	peripheral layer thickness
$\bar{a}$	non-dimensional quantity corresponding to a
f_2	frictional force at $y = h/2$
$\bar{F}_{2K}, \bar{F}_{2q}, \bar{F}_{2Q}$	Frictional force ratio parameters
h	film thickness
h_o	minimum film thickness
h_1	inlet film thickness
H, H_1	non-dimensional quantities corresponding to h and h_1 respectively
K	consistency ratio
m, m_1	consistency indices
n	flow behaviour index
p	pressure
q	thermal factor
Q	non-dimensional quantity corresponding to V
r	radius of the roller
R	radius of the equivalent roller
u, v	velocities along the coordinate axes
U	rolling velocity
V	normal velocity
$\bar{W}_K, \bar{W}_q, \bar{W}_Q$	non-dimensional load ratio parameters

CHAPTER 5

EFFECTS OF CONSISTENCY VARIATION OF POWER LAW LUBRICANTS FOR RIGID ROLLERS WITH ROUGH SURFACES

5.1 INTRODUCTION

In earlier chapters while studying the effects of consistency variation of power law lubricants on bearing characteristics, an implicit assumption was that the bearing surfaces were perfectly smooth. But, it has been recognized that owing to machining limitations this assumption is rather unrealistic. The topography of bearing surfaces is generally rough. It consists of a net work of randomly distributed micro-metallic structures forming 'hills and valleys' running on the surfaces. The study of these structures and their participation mechanism in the lubrication process is vital in determining the bearing characteristics, particularly when the dimensions of these structures are comparable with the thickness of the operating lubricant film.

Surface roughness in lubrication problems is studied by following deterministic and stochastic approaches. Burton [1] and Wildmann [2] studied the effects of surface roughness through deterministic approach by postulating a sine (cosine) wave or a series of sine (cosine) waves for the film thickness. Recently, a new deterministic theory has been proposed by Shukla [3] applicable when the mean height of the roughness structure is of the same

order as that of the minimum nominal (compliant) film thickness. This theory for hydrodynamic or mixed lubrication proposes a division of lubricant flow region into roughness interaction zone along the rough surface and purely hydrodynamic zone along the smooth surface. In the stochastic approach, the film thickness is assumed to be a random quantity varying with the roughness pattern. Christensen and Tonder [4] presented a stochastic theory for hydrodynamic lubrication of rough surfaces with roughness lay running along and transverse to the direction of motion, by making two heuristic assumptions about the pressure gradient and fluid flow rate. It was concluded that the type of roughness along the direction of motion results in a slight decrease in load carrying capacity and an increase in frictional force thereby causing a significant increase in coefficient of friction. The effect of the roughness structure transverse to the direction of motion is, however, to improve the bearing characteristics. Christensen [5] extended the stochastic theory to mixed lubrication regime. Raj and Sinha [6] employed Christensen's model to analyse the effect of roughness in short journal bearings assuming that the standard deviation of the distribution describing asperity heights and minimum nominal film thickness were of the same order. Rhow and Elrod [7] considered the spatial characteristics of roughness with respect to amplitude and wave length claiming some advantage

over stochastic analysis of Christensen and Tonder 'in both rigor and generality though not in brevity'. An average Reynolds equation for three dimensional isotropic and nonisotropic roughness structures has been derived by Patir and Cheng [8] by defining pressure flow factors obtained through flow simulation. Recently, Tonder [9] developed a roughness theory based on numerical evaluation of average flows and shear stresses.

The study of surface roughness together with the rheological considerations of the lubricant has been undertaken by some research workers. For e.g., Shukla and Kumar [1o] and Kumar and Sachidananda [11] considered effects of viscosity variation and surface roughness. Thermal effects in the lubrication of rough surfaces were studied by Dyson [12] and Fowles [13].

With the development of theoretical models to account for surface roughness in lubrication problem, a good deal of experimental work on surface roughness has been reported. Tsukada and Sasajima [14] developed a measuring system to investigate the three dimensional characteristics of asperities. An interferometic study of rough surfaces under ehd lubrication was conducted by Jackson and Cameron [15]. An extensive review of experimental report of various techniques for the study of separation of surfaces, the real area of contact,

the number of contact spots, the spatial distribution of the contact spots, the distribution of their sizes and the relation of all these to roughness and to the normal load was presented by Woo and Thomas [16].

Despite the development of various models to include the effects of surface roughness in lubrication problems, a critical analysis reveals that a most practical and general model is yet to emerge [17]. In reviewing the Christensen's model, Sun [18] questioned the validity of the heuristic assumptions made about pressure gradient and flow rate and pointed out the absence of roughness correlation terms in this model. Chen and Sun [17] remarked that Elrod's model [19] suffered from the restriction that the roughness spacing was small. The three dimensional roughness theory of Patir and Cheng [8] has the potentiality to describe the arbitrary roughness arrangement . However, the flow factors determined in this model are subject to alterations by the choice of numerical representation of contact, shape, size and boundary conditions of the model bearing, as has been demonstrated by Teale and Lebeck [20]. Despite the complexity or shortcomings of various models for surface roughness, the study of the rheological properties of the lubricant on rough bearings was attempted. Shukla and Kumar [21] considered viscosity variation and surface roughness in the analysis of slider

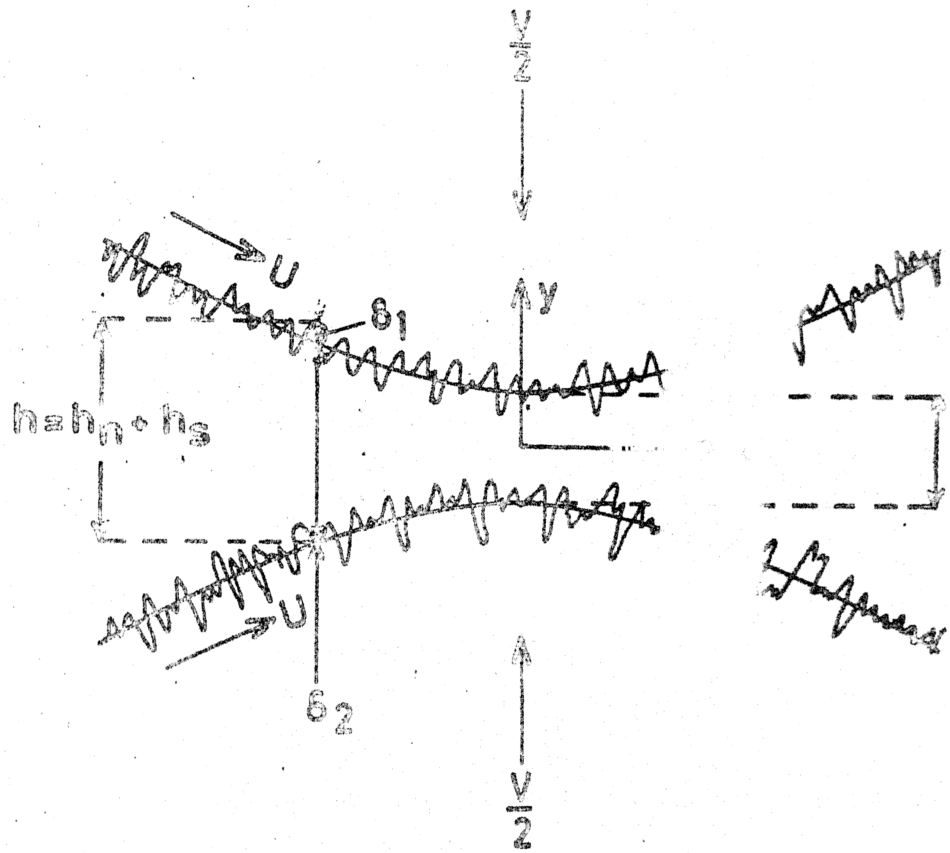
bearing lubrication using Newtonian fluid. Prakash et.al. [22] studied the roughness effects in slider bearing using a micropolar fluid and pointed out that the effects like enhancement of viscosity in thin film zones were rheological effects and might not be attributed to the surface roughness.

In this Chapter, we study the effects of surface roughness and consistency variation for rough rollers using a power law lubricant. Such a variation in consistency may be warranted by the reaction of additives and surfactants [23, 24]. Consistency variation is accounted through the model suggested in Chapter 2, while surface roughness is considered through Christensen's stochastic theory [25].

5.2 STOCHASTIC FORMS OF REYNOLDS EQUATION

The problem considered is that of the lubrication of two identical rollers each of radius r, lubricated with a power law fluid (Fig.5.1). U is the rolling velocity and V/2 is the normal velocity of the rollers. The rollers are separated by a film thickness h which is considered to be a stochastic variable depending on the roughness structure of the bearing surface. Further, we assume the conditions for applicability of Reynolds equation for treating rough surfaces [17 i.e.,

(1) the roughness spacing is large compared with the film thickness, and



$h_s = \delta_1 + \delta_2$

Fig. 5.1: A schematic diagram of a rough roller bearing.

(2) the roughness height is small compared with the film thickness so that no recirculation region forms in the film.

As pointed out earlier, the bearing surfaces are generally rough. To take this into account, the film thickness is to be viewed in the light of surface asperities in the lubricated region. Stochastically, the film thickness is expressed as [25]

$$h = h_n(x,z,t) + h_s(x,z,\xi) \tag{5.1}$$

where h_n denotes the nominal part of the film geometry and h_s is the roughness part measured from the nominal level. ξ appearing in h_s is a random variable describing the roughness arrangement. Assigning a particular value to ξ means selecting a particular arrangement from the supply of possible roughness arrangements. For a given value of ξ, the surface roughness component h_s is a deterministic function of the space variables. We consider for our study one dimensional roughness pattern formed in the lubricating machinery, particularly after they have received some run-in and wear [6,24]. Keeping note of this, the film thickness in the case of rollers can be written as [8]

$$h = h_n + \delta_1 + \delta_2 \tag{5.2}$$

where δ_1 and δ_2 follow Gaussian distribution function with mean zero and combined standard deviation σ.

For many engineering surfaces the use of Gaussian function to describe the roughness profile at the vertical scale is valid at least upto three standard deviations[26]. For mathematical simplicity but with a fair degree of accuracy, a polynomial function which closely approximates Gaussian function [27] is employed in the undergoing analysis :

$$\begin{aligned} f(h_s) &= ((35/32)/b^7)(b^2-h_s^2)^3 \qquad -b < h_s < b \\ &= 0 \qquad\qquad\qquad\qquad\qquad\quad \text{otherwise} \end{aligned} \tag{5.3}$$

where the relationship between the roughness amplitude b measured from nominal level and standard deviation σ is given by $b = \pm 3\sigma$.

In the following section, we shall derive stochastic form of Reynolds equation considering cavitation, applicable to rough rolling surfaces, using Christensen's approach [24]. For this purpose, the expected values of various quantities may be defined as follows :

$$E(\;\;) = \int_{-\infty}^{\infty} (\;\;)\, f(h_s)\,dh_s \tag{5.4}$$

where $f(h_s)$ is the probability density function of the random variable h_s.

If the function is assumed to be symmetric, then

$$\int_{-\infty}^{\infty} h_s^{2r+1} f(h_s) dh_s = 0 \quad r = 0,1,2,.. \tag{5.5}$$

This assumption is valid since the mean of h_s over the bearing surface is zero and can be taken as the definition of nominal film thickness [24]. In particular, for the ergodic (stationary) stochastic process, we have

$E(h) = h_n$.

With the help of eqns. (2.10) and (2.11) and Fig.5.1 the basic equations determining pressure are written as

$$\frac{d}{dx}\{(f)(\frac{dp_1}{dx})^{1/n}\} = -\frac{V}{2} + \frac{U}{2}\frac{dh}{dx} \qquad -\infty \leq x \leq -x^* \tag{5.6}$$

$$\frac{d}{dx}\{(f)(-\frac{dp_2}{dx})^{1/n}\} = -(-\frac{V}{2} + \frac{U}{2}\frac{dh}{dx}) \quad -x^* \leq x \leq x_c \tag{5.7}$$

where
$$(f) = \int_0^{h/2} y(y/m)^{1/n} dy \tag{5.8}$$

and $-x^*$ and x_c are the points of maximum pressure and cavitation respectively; p_1 and p_2 denote the pressure p in the regions defined in eqns. (5.6) and (5.7) where pressure gradients are positive and negative respectively.

Considering consistency variation, the consistency m is given by

$$\begin{aligned} m &= m_1(h/h_1)^q \qquad 0 \leq y \leq h_n/2 - a_p \\ &= Km_1(h/h_1)^q \quad h_n/2 - a_p < y \leq h/2 \end{aligned} \tag{5.9}$$

where a_p is the apparent peripheral layer measured from the nominal level, h_1 is inlet film thickness where q is measured. Substituting the expression for m from eqn. (5.9) in eqn. (5.8) we obtain

$$(f) = (n/(2n+1))(1/2)^{(2n+1)/n} \; h_1^{q/n} \; (1/m_1)^{1/n} (f_r) \quad (5.10)$$

where $(f_r) = \{(1-K^{-1/n})(h_n-2a_p)^{(2n+1)/n} + K^{-1/n} \, h^{(2n+1)/n}\} / \, h^{q/n}$

(5.11)

It is clear that for the smooth case,(f) in eqn. (5.10) is reduced to that obtained in eqn. (2.15).

(i) ONE DIMENSIONAL LONGITUDINAL ROUGHNESS

In this type of roughness, the lay of roughness structure consists of long narrow ridges and valleys running parallel to the rolling direction [24] (Fig.5.2(a)). Thus, the roughness components δ_1 and δ_2 are independent of x, U and t (time) [28]. We note that the pressure gradient $\frac{dp}{dx}$ is a variable with zero or negligible variance and we have [25], [29]

$$E(\pm \frac{dp}{dx})^{1/n} = (\pm \frac{dE(p)}{dx})^{1/n} \quad (5.12)$$

Now, taking expectation on both sides of eqns. (5.6 and (5.7), we obtain the stochastic form of Reynolds equation applicable to rollers with longitudinal surface roughness :

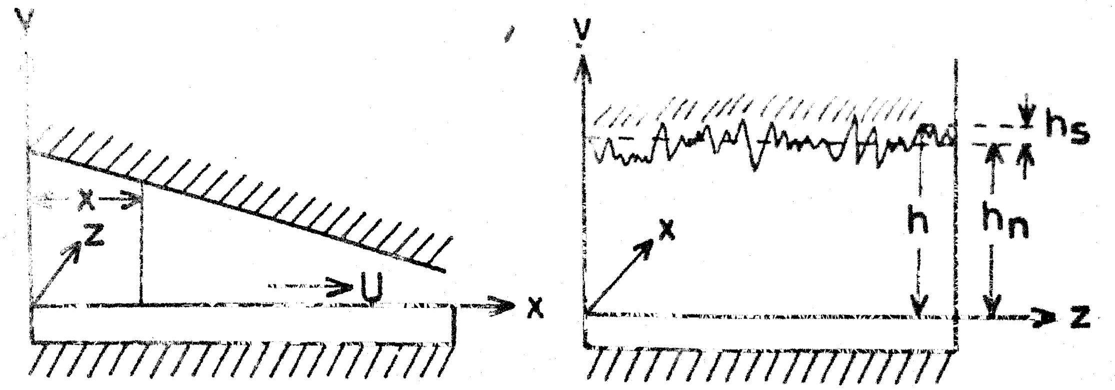


Fig. 5.2 (a) Longitudinal roughness.

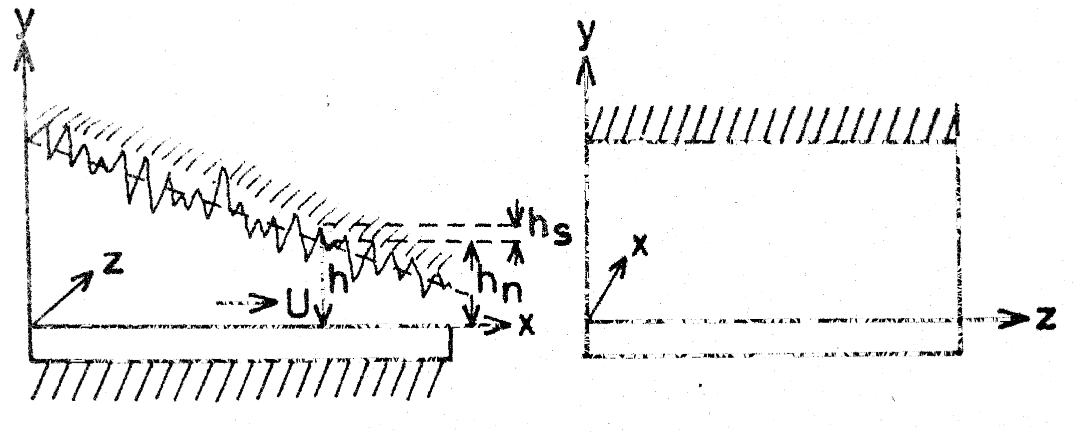


Fig. 5.2 (b) Transverse roughness.

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} E(f_r)\left(\frac{1}{m_1}\frac{dE(p_1)}{dx}\right)^{1/n}\right]$$

$$= U\frac{\partial h_n}{\partial x} + \frac{\partial h_n}{\partial t} \qquad -\infty \leq x \leq -x^* \tag{5.13.}$$

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} E(f_r)\left(-\frac{1}{m_1}\frac{dE(p_2)}{dx}\right)^{1/n}\right]$$

$$= -\left(U\frac{\partial h_n}{\partial x} + \frac{\partial h_n}{\partial t}\right) \qquad -x^* \leq x \leq x_c \tag{5.14}$$

where $\frac{\partial h_n}{\partial t} = -V$. The nominal film thickness h_n is given by

$$h_n = h_o + x^2/(2R) \tag{5.15}$$

where h_o is the nominal minimum film thickness and R is the radius of the equivalent roller.

In this Chapter, we study the effect of roughness on rigid rolling surfaces only. For this purpose, we have to solve the following equations to obtain average pressure E(p):

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} E(f_r)\left(\frac{1}{m_1}\frac{dE(p_1)}{dx}\right)^{1/n}\right] = U\frac{dh_n}{dx}$$

$$-\infty \leq x \leq -x^* \tag{5.16}$$

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} E(f_r)\left(-\frac{1}{m_1}\frac{dE(p_2)}{dx}\right)^{1/n}\right] = -U\frac{dh_n}{dx}$$

$$-x^* \leq x \leq x_c \tag{5.17}$$

We use the following boundary conditions :

$$E(p_1) = 0 \quad \text{at } x = -\infty \tag{5.18}$$

$$\frac{dE(p_1)}{dx} = \frac{dE(p_2)}{dx} = 0 \text{ at } x = -x^* \qquad (5.19)$$

$$E(p_1) = E(p_2) \qquad \text{at } x = -x^* \qquad (5.20)$$

$$E(p_2) = \frac{dE(p_2)}{dx} = 0 \text{ at } x = x_c \qquad (5.21)$$

Integrating eqns. (5.16) and (5.17) with condition (5.19) we get

$$\frac{dE(p_1)}{dx} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \frac{(h_n-\overset{*}{h}_n)^n}{\{E((f_r))\}^n}$$

$$-\infty \leq x \leq -x^* \qquad (5.22)$$

$$\frac{dE(p_2)}{dx} = -\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \frac{(\overset{*}{h}_n-h_n)^n}{\{E((f_r))\}^n}$$

$$-x^* \leq x \leq x_c \qquad (5.23)$$

where $h_n = \overset{*}{h}_n$ at $x = -x^*$.

Using eqns. (5.15) and (5.21) in eqn. (5.23) we obtain $x_c = x^*$.

Integrating again eqns. (5.22) and (5.23), with conditions (5.18), (5.20) and (5.21) we obtain the average pressure given by

$$E(p_1) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{x} \frac{(h_n-\overset{*}{h}_n)^n}{\{E((f_r))\}^n} dx$$

$$-\infty \leq x \leq -x^* \qquad (5.24)$$

$$E(p_2) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-x^*} \frac{(h_n-h_n^*)^n}{\{E((f_r))\}^n} dx - \int_{-x^*}^{x} \frac{(h_n^*-h_n)^n}{\{E((f_r))\}^n} dx \right]. \qquad (5.25)$$

Using condition (5.21) in eqn. (5.25), we get the equation to determine cavitation point :

$$\int_{-\infty}^{-X^*} I_{L1}(n,K,q,B)\, dX = \int_{-X^*}^{X^*} I_{L2}(n,K,q,B)\, dX \qquad (5.26)$$

where

$$I_{L1}(n,K,q,B) = (H_n-H_n^*)^n / \{E((F_r))\}^n \qquad (5.27)$$

$$I_{L2}(n,K,q,B) = (H_n^*-H_n)^n / \{E((F_r))\}^n \qquad (5.28)$$

$$E((F_r)) = \frac{35}{32B^7} \int_{-B}^{B} (B^2-Hs^2)^3 (F_r)\, dHs \qquad (5.29)$$

$$(F_r) = \left[(1-K^{-1/n})(H_n-2\bar{a}_p)^{(2n+1)/n} + K^{-1/n}(H_n+H_s)^{(2n+1)/n}\right] / (H_n+H_s)^{q/n} \qquad (5.30)$$

$X = x/\sqrt{(2Rh_o)}$, $X^* = x^*/\sqrt{(2Rh_o)}$, $H = H_n+H_s$, $H = h/h_o$, $H_s = h_s/h_o$

$H_n = h_n/h_o = 1+X^2$, $H_n^* = h_n^*/h_o = 1+X^{*2}$, $\bar{a}_p = a_p/h_o$,

$B = b/h_o$. (5.31)

The load component $W_{Lx}(n,K,q,b)$ per unit length in the x direction for the case of longitudinal roughness is given by

$$W_{Lx}(n,K,q,b) = \int_{-\infty}^{-x^*} x^2 \frac{dE(p_1)}{dx} dx + \int_{-x^*}^{x^*} x^2 \frac{dE(p_2)}{dx} dx \qquad (5.32)$$

which on using eqns. (5.22) and (5.23) becomes

$$W_{Lx}(n,K,q,b) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 \frac{U^n}{R} \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-x^*} \frac{x^2 (h_n - h_n^*)^n}{\{E(f_r)\}^n} dx - 2 \int_0^{x^*} \frac{x^2 (h_n^* - h_n)^n}{\{E(f_r)\}^n} dx \right] \tag{5.33}$$

We obtain the load component $W_{Ly}(n,K,q,b)$ along y direction is given by

$$W_{Ly}(n,K,q,b) = -\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-x^*} \frac{x (h_n - h_n^*)^n}{\{E(f_r)\}^n} dx \tag{5.34}$$

To study the effects of longitudinal roughness and consistency variation on load, we calculate the load ratios $\bar{W}_{LX}$ and $\bar{W}_{LY}$ defined as follows :

$$\bar{W}_{LX}(n,K,q,B) = \frac{W_{Lx}(n,K,q,b) - W_{Sx}(n,K,q)}{W_{Sx}(n,K,q)}$$

$$= \frac{\int_{-\infty}^{-X^*} X^2 I_{L1}(n,K,q,B)\,dX - 2\int_0^{X^*} X^2 I_{L2}(n,K,q,B)\,dX}{\int_{-\infty}^{-X_S^*} X^2 I_{S1}(n,K,q)\,dX - 2\int_0^{X_S^*} X^2 I_{S2}(n,K,q)\,dX} - 1 \tag{5.35}$$

$$\bar{W}_{LY}(n,K,q,B) = \frac{W_{LY}(n,K,q,b) - W_{Sy}(n,K,q)}{W_{Sy}(n,K,q)} - 1$$

$$= \frac{\int_{-\infty}^{-X^*} XI_{L1}(n,K,q,B)dX}{\int_{-\infty}^{-X_S^*} XI_{S1}(n,K,q)dX} - 1 \qquad (5.36)$$

where $W_{Sx}(n,K,q)$ and $W_{Sy}(n,K,q)$ are given by (eqns. (2.36) and (2.38))

$$W_{Sx}(n,K,q) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} \frac{m_1}{R} U^n \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-x_S^*} \frac{x^2(h_n-h_n^*)^n}{(f_o)^n h_n^{2n+1-q}} dx - 2\int_0^{x_S^*} \frac{x^2(h_n^*-h_n)^n}{(f_o)^n h_n^{2n+1-q}} dx\right] \qquad (5.37)$$

and

$$W_{Sy}(n,K,q) = -\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-x_S^*} \frac{x(h_n-h_n^*)^n}{(f_o)^n h_n^{2n+1-q}} dx \qquad (5.38)$$

where $-x_S^*$ and x_S^* are points of maximum pressure and cavitation, respectively, for smooth case and $I_{S1}(n,K,q)$ and $I_{S2}(n,K,q)$ are given by

$$I_{S1} = (H_n-H_n^*)^n / \{(F_o)^n H_n^{2n+1-q}\} \qquad (5.39)$$

$$I_{S2} = (H_n^*-H_n)^n / \{(F_o)^n H_n^{2n+1-q}\} \qquad (5.40)$$

(F_o) being given by eqn. (2.33). The cavitation point X_S^* in the smooth case can be obtained from eqn. (2.31).

(ii) ONE DIMENSIONAL TRANSVERSE ROUGHNESS

In this case, the roughness lay consists of long narrow ridges and valleys transverse to the direction of motion [25] (Fig.5.2(b)). This type of roughness pattern may be generated in bearings subject to oscillations perpendicular to the principal motion [6]. The film thickness in the case of transverse roughness is given by [27]

$$h = h_n + \delta_1(x-Ut) + \delta_2(x-Ut). \tag{5.41}$$

Noting that $\frac{\partial}{\partial t}\delta_i(x-Ut) = -U\frac{\partial \delta_i}{\partial x}$ $i = 1,2,$ (5.42)

and
$$\frac{\partial h}{\partial x} = \frac{\partial h_n}{\partial x} + \frac{\partial \delta_1}{\partial x} + \frac{\partial \delta_2}{\partial x} \tag{5.43}$$

eqns. (5.6) and (5.7) become

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\,(f_r)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n} - Uh_n\right] = \frac{\partial h_n}{\partial t}$$

$$-\infty \leq x \leq -x^* \tag{5.44}$$

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\,(f_r)\left(-\frac{1}{m_1}\frac{dp_2}{dx}\right)^{1/n} + Uh_n\right] = -\frac{\partial h_n}{\partial t}$$

$$-x^* \leq x \leq x_c. \tag{5.45}$$

We denote the square bracketed term in eqn. (5.44) by M in accordance with reference [28]:

$$M = \frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\,(f_r)\left(\frac{1}{m_1}\frac{dp_1}{dx}\right)^{1/n} - Uh_n. \tag{5.46}$$

On rearranging, we have

$$\frac{(M+Uh_n)^n}{(f_r)^n} = \left(-\frac{2n}{2n+1}\right)^n \left(\frac{1}{2}\right)^{2n+1} h_1^q \frac{1}{m_1} \frac{dp_1}{dx} \tag{5.47}$$

In this case, M is a stochastic variable with zero or negligible variance. Hence, we have [29]

$$E(M+Uh_n)^n = \{E(M)+Uh_n\}^n = (M+Uh_n)^n \tag{5.48}$$

Taking expectation for terms in eqn. (5.44) using eqns. (5.46) and (5.48) we get

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} (h_1)^{q/n} \left\{E\left(\frac{1}{(f_r)^n}\right)\right\}^{-1/n} \left(\frac{1}{m_1}\frac{dE(p_1)}{dx}\right)^{1/n}\right]$$

$$= U\frac{\partial h_n}{\partial x} + \frac{\partial h_n}{\partial t} \qquad -\infty \leq x \leq -x^* \tag{5.49}$$

Similarly taking expectation, eqn. (5.45) becomes

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} (h_1)^{q/n} \left\{E\left(\frac{1}{(f_r)^n}\right)\right\}^{-1/n} \left(-\frac{1}{m_1}\frac{dE(p_2)}{dx}\right)^{1/n}\right]$$

$$= -\left(U\frac{\partial h_n}{\partial x} + \frac{\partial h_n}{\partial t}\right) \qquad -x^* \leq x \leq x_c \tag{5.50}$$

The average Reynolds equation for the case of transverse roughness under pure rolling conditions becomes

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} (h_1)^{q/n} \left\{E\left(\frac{1}{(f_r)^n}\right)\right\}^{-1/n} \left(\frac{1}{m_1}\frac{dE(p_1)}{dx}\right)^{1/n}\right]$$

$$= U\frac{dh_n}{dx} \qquad -\infty \leq x \leq -x^* \tag{5.51}$$

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n}(h_1)^{q/n}\left\{E\left(\frac{1}{(f_r)^n}\right)\right\}^{-1/n}\left(-\frac{1}{m_1}\frac{dE(p_2)}{dx}\right)^{1/n}\right]$$

$$= -U\frac{dh_n}{dx} \qquad -x^* \leq x \leq x_c \qquad (5.52)$$

Following the same procedure as adopted in the case of longitudinal roughness, the expressions for $E(p_1)$ and $E(p_2)$ can be obtained as

$$E(p_1) = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1}\left(\frac{1}{h_1}\right)^q U^n m_1 \int_{-\infty}^{x}(h_n-h_n^*)^n E\left(\frac{1}{(f_r)^n}\right)dx$$

$$-\infty \leq x \leq -x^* \qquad (5.53)$$

$$E(p_2) = -\left(\frac{2n+1}{2n}\right)^n 2^{2n+1}\left(\frac{1}{h_1}\right)^q U^n m_1\left[\int_{-\infty}^{-x^*}(h_n-h_n^*)^n E\left(\frac{1}{(f_r)^n}\right)dx\right.$$

$$\left.-\int_{-x^*}^{x}(h_n^*-h_n)^n E\left(\frac{1}{(f_r)^n}\right)dx\right] \quad -x^* \leq x \leq x^* \qquad (5.54)$$

The point of cavitation X^* is obtained from the eqn.

$$\int_{-\infty}^{-X^*} I_{T1}(n,K,q,B)\,dX = \int_{-X^*}^{X^*} I_{T2}(n,K,q,B)\,dX \qquad (5.55)$$

where $$I_{T1}(n,K,q,B) = (H_n-H_n^*)^n E\left(\frac{1}{(F_r)^n}\right) \qquad (5.56)$$

$$I_{T2}(n,K,q,B) = (H_n^*-H_n)^n E\left(\frac{1}{(F_r)^n}\right) \qquad (5.57)$$

The load components $W_{Tx}(n,K,q,b)$ and $W_{Ty}(n,K,q,b)$ are given by

$$W_{Tx}(n,K,q,b)$$

$$=\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 \frac{U^n}{R}\left(\frac{1}{h_1}\right)^q\left[\int_{-\infty}^{-x^*} x^2(h_n-h_n^*)^n E\left(\frac{1}{(f_r)^n}\right)dx\right.$$

$$\left.-2\int_0^{x^*} x^2(h_n^*-h_n)^n E\left(\frac{1}{(f_r)^n}\right)dx\right] \qquad (5.58)$$

and

$$W_{Ty}(n,K,q,b)=$$

$$-\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_1 U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{x^*} x(h_n-h_n^*)^n E\left(\frac{1}{(f_r)^n}\right)dx$$

(5.59)

As in the case of longitudinal roughness, we define the load ratios $\bar{W}_{TX}$ and $\bar{W}_{TY}$ to study the effects of consistency variation and roughness for transverse case:

$$\bar{W}_{TX}(n,K,q,B) = \frac{W_{Tx}(n,K,q,b)-W_{Sx}(n,K,q)}{W_{Sx}(n,K,q)}$$

$$= \frac{\int_{-\infty}^{-X^*} X^2 I_{T1}(n,K,q,B)\,dX - 2\int_0^{X^*} X^2 I_{T2}(n,K,q,B)\,dX}{\int_{-\infty}^{-X_S^*} X^2 I_{S1}(n,K,q)\,dX - 2\int_0^{X_S^*} X^2 I_{S2}(n,K,q)\,dX} - 1$$

(5.60)

$$\bar{W}_{TY}(n,K,q,B) = \frac{W_{Ty}(n,K,q,b)-W_{Sy}(n,K,q)}{W_{Sy}(n,K,q)}$$

$$= \frac{\int_{-\infty}^{-X^*} X I_{T1}(n,K,q,B)\,dX}{\int_{-\infty}^{-X_S^*} X I_{S1}(n,K,q)\,dX} - 1 \qquad (5.61)$$

In the absence of consistency variation (i.e. $K = 1$ and $q = 0$) the load ratio quantities for longitudinal and transverse roughness reduce to those in the reference [30].

5.3 RESULTS AND DISCUSSION

While dividing the operating lubricant film into central layer and peripheral layer with different consistencies, we defined in eqn. (5.9) apparent peripheral layer a_p. This layer is to be distinguished from the actual peripheral layer 'a' which is the sum of the apparent peripheral layer and the roughness part of the film thickness from the mean level on each surface. Precisely ,'a' is stochastic and 'a_p' is deterministic quantities. It is obvious that it is the actual peripheral layer which takes part in the lubrication process.

In the calculation procedure, the average pressure is obtained by using a symmetric function which approximates Gaussian function. This necessitates the roughness amplitude b/2 at each rolling surface to be less than or equal to a_p (i.e. $a_p \geq b/2$). This enables the 'hills and valleys' to be completely encompassed by the apparent peripheral layer. Since the roughness heights are measured from the mean level, a_p may be measured from the mean level. In the theoretical analysis, to study the effect of consistency variation across the film

thickness, a_p can be regarded as a parameter.

To study the effect of longitudinal roughness and consistency variation, the cavitation points X^* for the longitudinal case and X_S^* for the smooth case are determined from eqns. (5.26) and (2. 31) and the load ratios $\bar{W}_{LX}$ and $\bar{W}_{LY}$ are evaluated using eqns. (5.35) and (5.36) respectively. Similarly X^* for transverse case can be determined from eqn. (5.55) and load ratios $\bar{W}_{TX}$ and $\bar{W}_{TY}$ can be evaluated using eqns (5.60) and (5.61) respectively. Table 1 gives the picture of the effect of roughness parameter B on the load ratios $\bar{W}_{LX}$ and $\bar{W}_{TX}$ for various values of n for the cases $K > 1$ and $K < 1$. Table 2 gives the picture of load ratios $\bar{W}_{LY}$ and $\bar{W}_{TY}$ for different values of B. It is seen from the Tables 1 and 2 that the roughness effect is to decrease the load capacity in the longitudinal case and increase it in the transverse case for all n for any fixed q and K. The results obtained are in good qualitative agreement with those of Sinha et.al. [30]. In the case of longitudinal roughness, for $K < 1$, there is slight increase in load capacity as compared to the case of $K > 1$ for all n. It is evident that along the direction of motion the hydrodynamic action of the lubricant contributes to the load carrying capacity . In the longitudinal case, both for $K > 1$ and $K < 1$, the peripheral layer is attached to the bearing surface and the lubrication process depends mainly on the consistency of the central layer. If the consistency in this layer is higher than

Table 1

Rough parameter B vs. load ratios $\bar{W}_{LX}$ and $\bar{W}_{TX}$ for various values of n, K > 1 and K < 1

	K = 1.5						K=0.75					
	Longitudinal			Transverse			Longitudinal			Transverse		
B	n=0.5 10^{-3}x	n=1.0 10^{-3}x	n=1.5 10^{-3}x	n=0.5 10^{-2}x	n=1.0 10^{-2}x	n=1.5 10^{-2}x	n=0.5 10^{-3}x	n=1.0 10^{-3}x	n=1.5 10^{-3}x	n=0.5 10^{-2}x	n=1.0 10^{-2}x	n=1.5 10^{-2}x
.1	-0.06	-0.13	-0.25	0.00	0.02	0.08	-0.05	-0.09	-0.18	0.00	0.02	0.06
.2	-0.24	-0.52	-1.00	0.02	0.11	0.32	-0.19	-0.37	-0.73	0.02	0.09	0.26
.3	-0.54	-1.16	-2.25	0.06	0.26	0.74	-0.42	-0.84	-1.63	0.05	0.20	0.59

Table 2

Roughness parameter B vs. load ratios $\bar{W}_{TX}$ and $\bar{W}_{TY}$ for various values of n , K > 1 and K < 1

	K = 1.5						K = 0.75					
	Longitudinal			Transverse			Longitudinal			Transverse		
B	n=0.5 10^{-3}x	n=1.0 10^{-3}x	n=1.5 10^{-3}x	n=0.5 10^{-2}x	n=1.0 10^{-2}x	n=1.5 10^{-2}x	n=0.5 10^{-3}x	n=1.0 10^{-3}x	n=1.5 10^{-3}x	n=0.5 10^{-2}x	n=1.0 10^{-2}x	n=1.5 10^{-2}x
.1	-0.19	-0.33	-0.50	0.02	0.07	0.15	-0.13	-0.23	-0.37	0.01	0.05	0.13
.2	-0.76	-1.30	-1.99	0.08	0.29	0.64	-0.52	-0.42	-0.14	0.06	0.22	0.52
.3	-1.70	-2.91	-4.44	0.20	0.67	1.46	-0.11	-2.06	-3.27	0.15	0.51	1.20

that of the peripheral layer there is an increase in the load capacity. Hence $K < 1$ brings in an increase in the load capacity.

In the case of transverse roughness, for $K > 1$, the high consistency fluid entrapped between the asperities in dominant providing resistance to motion, which in turn increases the load capacity. This dominance is supplemented by the predominant squeezing motion of the fluid in the transverse case and by the extensive length of the bearing in the axial direction. Hence we have the increase in load capacity for $K > 1$ for transverse roughness.

Considering no consistency variation across the film thickness for the smooth case, evaluation of the load ratios enable us to study the combined effect of peripheral consistency variations and surface roughness which is depicted in Fig.5.3. It is observed that as K increases, the load capacity increases for all n. The trend is similar to that of the smooth case discussed in Chapter 2. However, for some value of K lying between 0.4 and 0.7, this trend is violated i.e. there is slight decrease in the load ratio for $n = 0.5$ compared to $n = 1.0$ for $K < 1$. Thus we see that there exists a critical value of K (say, K_c) which may depend on the flow behaviour index n as well as the roughness parameter B below which the load ratio trend is different from that of the smooth case. The effect of the rheological anomalies could be complex for these values

of $K < K_c$ where no set pattern for flow behaviour index can be predicted. This is elaborated in Table 3 where the load ratio $\bar{W}_{LX}$ and $\bar{W}_{LY}$ are tabulated against different n for fixed $K(=0.4)$.

We observe an increase in the load ratio when n decreases from 0.3 to 0.7. It probably reaches a maximum at some value of n lying between 0.7 and 1.1 and then starts decreasing. This deviation in the load ratios can be attributed to the complex nature of the rheological anomalies in the presence of asperities. Similar behaviour is observed in the transverse case as well.

Table 3

The load ratios $\bar{W}_{LX}$ and $\bar{W}_{LY}$ vs. flow behaviour index n

n	$\bar{W}_{LX}$	$\bar{W}_{LY}$
.3	.743	.744
.7	.840	.840
1.1	.827	.828
1.5	.798	.799
1.9	.770	.771

$K=0.4$ $a_p=0.3$ $B=0.2$ $q=0.5$

In the case of transverse roughness, similar interpretations (as advanced for Fig.5.3) can be made for the load ratios $\bar{W}_{TX}$ and $\bar{W}_{TY}$ (Fig.5.4).

In Fig. 5.5 is depicted the effect of K on $\bar{W}_{LX}$ and $\bar{W}_{LY}$ for various values of q. For K < 1 as q increases, the load ratios increases for all n and for K > 1 load capacity decreases for increasing q low values of q effect the load ratios $\bar{W}_{Lx}$ and $\bar{W}_{LY}$ significantly for any K. Similar interpretations can be given for the case of transverse roughness (Fig.5.6).

5.4 CONCLUSION

With the consideration of consistency variation across the film thickness, the effect of the longitudinal roughness is to reduce the load capacity for all n and transverse roughness is to increase it. Because of the interaction of the peripheral layer with the surface asperities, there exists a critical value of the consistency ratio above which the pattern of the flow behaviour index is similar to that of the consistency variation in the smooth case. When the consistency is higher in the central layer, there is greater load capacity for longitudinal roughness. However when the consistency is higher in the peripheral region in the transverse case, there is greater protection against seizure and load capacity. With the given consistency variation parameter across the film, the effect of the thermal factor is to increase the load ratio for a decrease of consistency in the peripheral region and decrease the load ratio for increasing consistency in the peripheral region for all values of n.

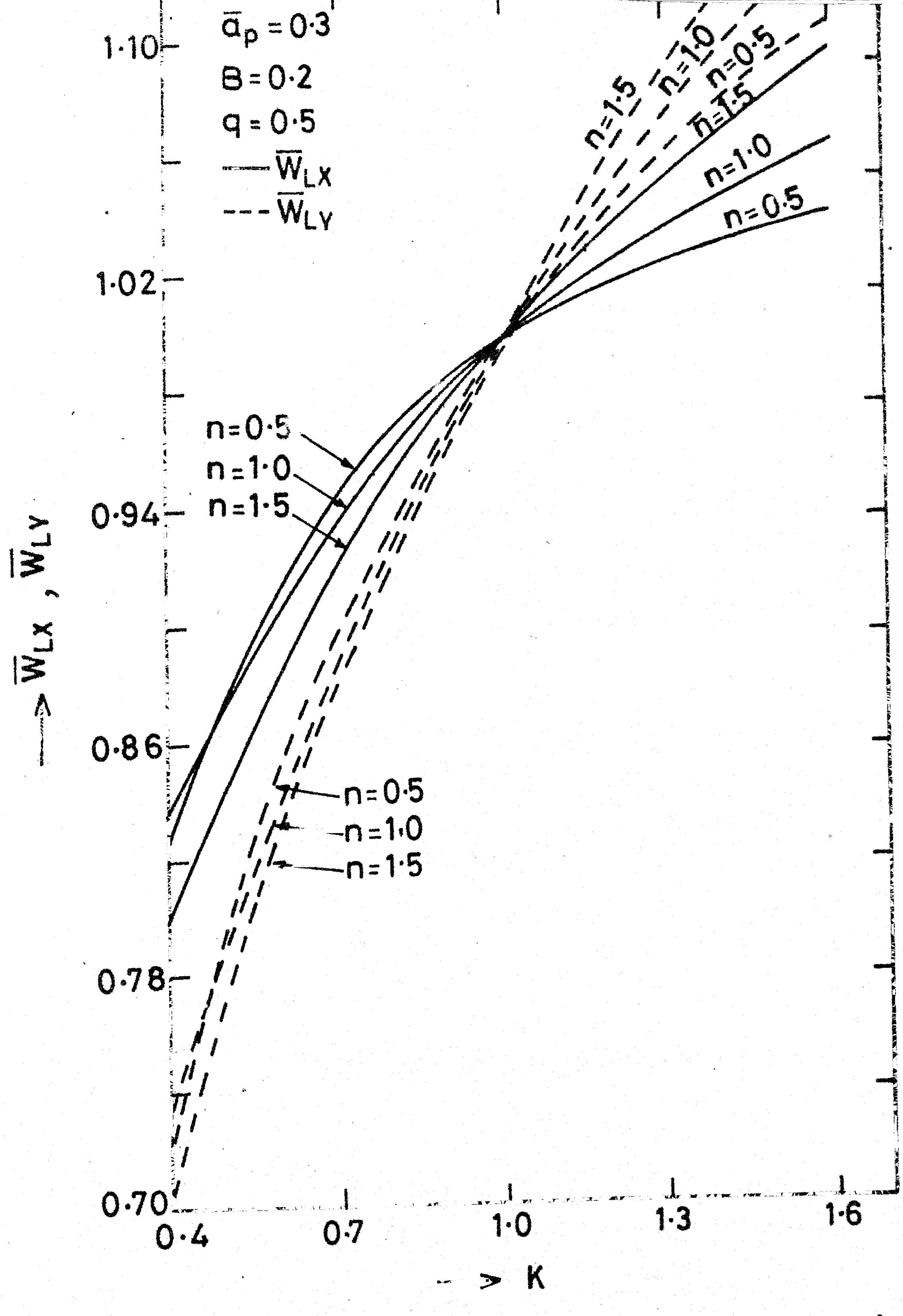


Fig. 5·3 Effect of consistency ratio K on load ratios $\overline{W}_{LX}$ and $\overline{W}_{LY}$ for various values of flow behaviour index n

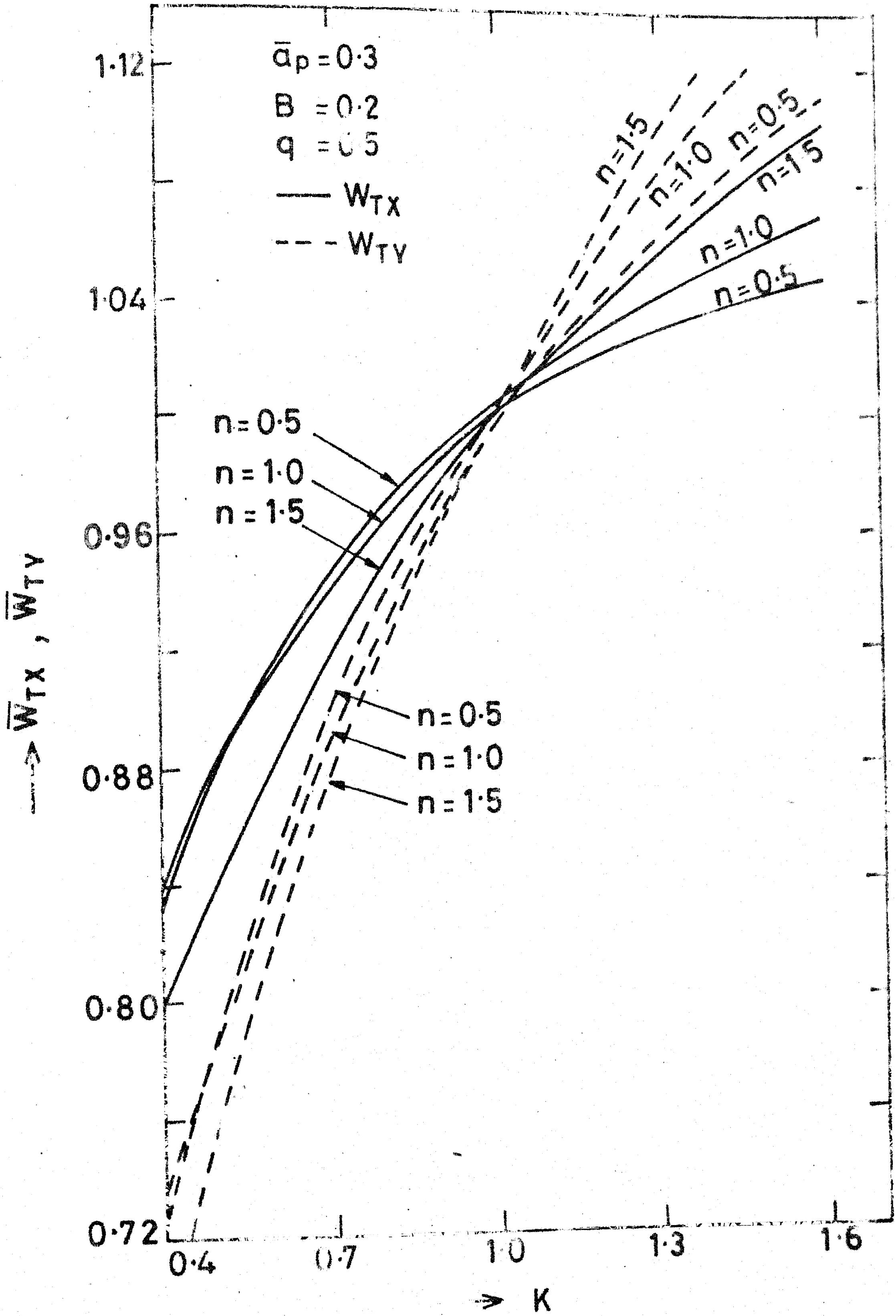


Fig.5·4 Effect of consistency ratio K on load ratios $\overline{W}_{TX}$ and $\overline{W}_{TY}$ for various values of flow behaviour index n

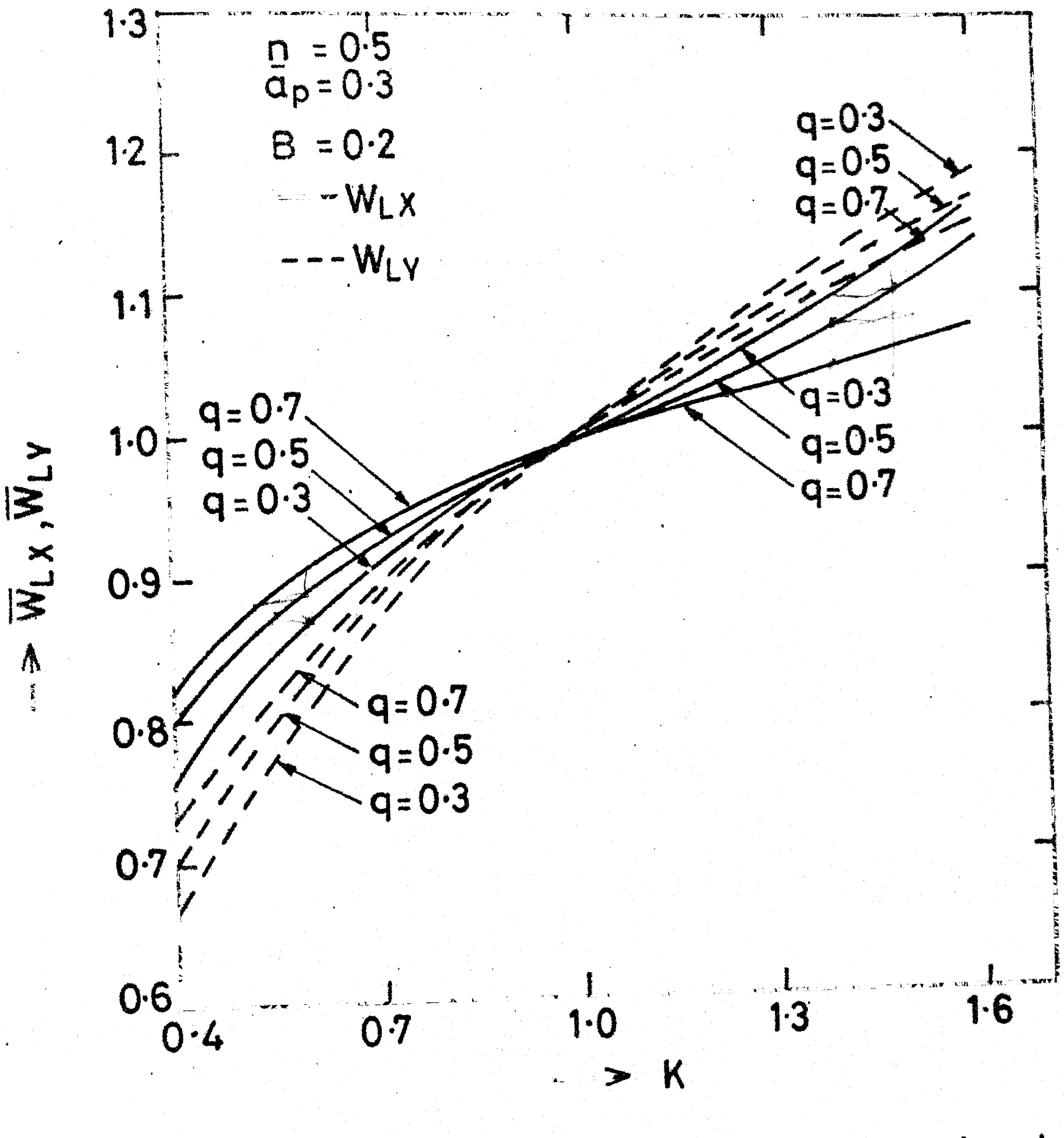


Fig.5·5 Effect of consistency ratio K on load ratios $\overline{W}_{LX}, \overline{W}_{LY}$ for various values of thermal factor q

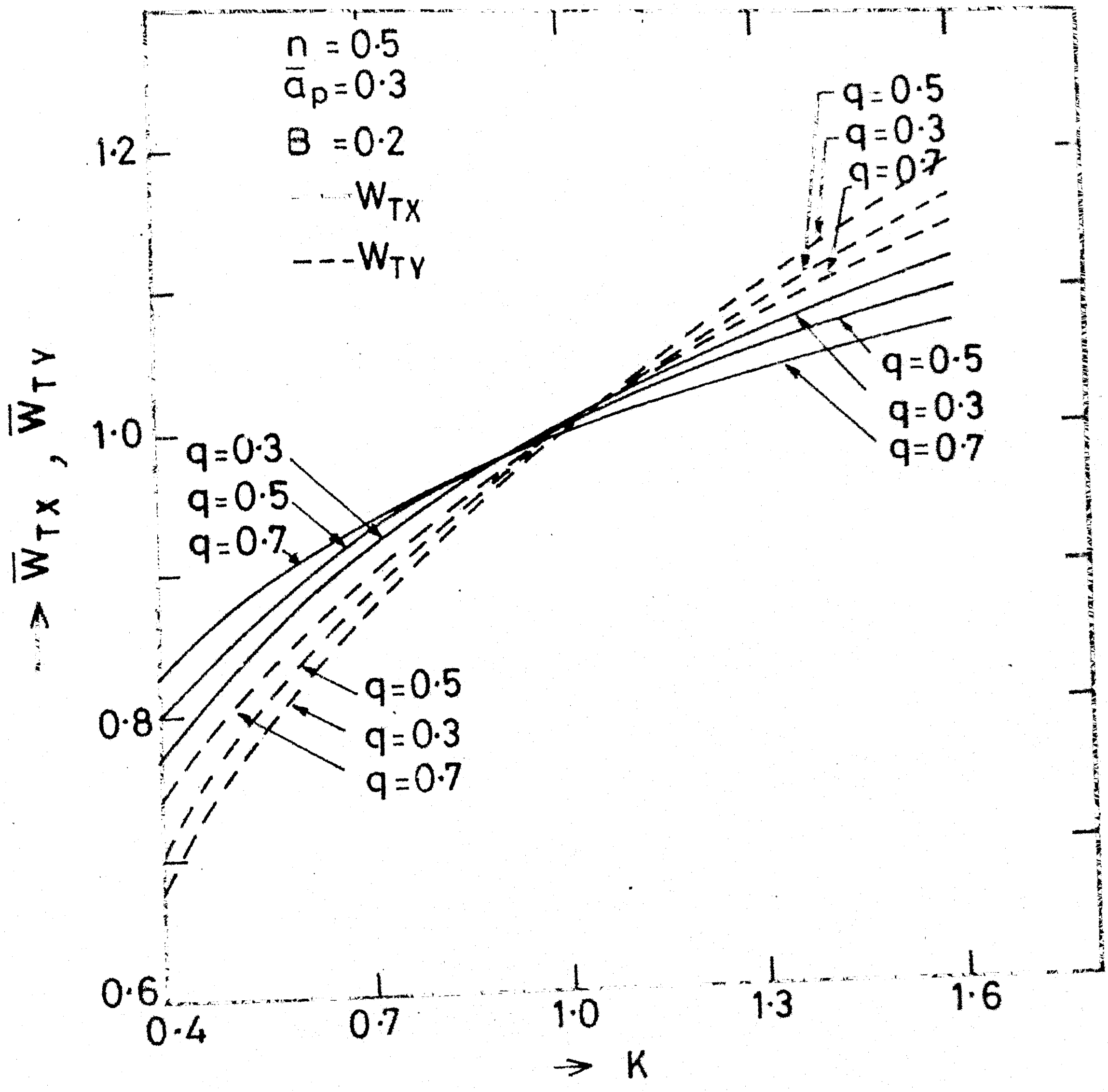


Fig. 5·6 Effect of consistency ratio K on load ratios $\overline{W}_{TX}$ and $\overline{W}_{TY}$ for various values of thermal factor q

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NOMENCLATURE

a_p	apparent peripheral layer
b	roughness amplitude
$E(\ )$	expectation of ( )
h	film thickness
h_n	nominal part of film thickness
h_s	roughness part of film thickness
K	consistency ratio
m, m_1	consistency indices
n	flow behaviour index
p, p_1, p_2	hydrodynamic pressures
q	thermal factor
r	radius of the roller
R	radius of the equivalent roller
t	time
U	rolling velocity
u, v	velocities along the coordinate axes
W_{Lx}, W_{Ly}	loads in the longitudinal roughness
W_{Tx}, W_{Ty}	loads in the transverse roughness
W_{Sx}, W_{Sy}	loads for the smooth case
$\bar{W}_{LX}, \bar{W}_{LY}, \bar{W}_{TX}, \bar{W}_{TY}$	load ratios

CHAPTER 6

EFFECTS OF CONSISTENCY VARIATION ON SQUEEZE FILMS BETWEEN ROUGH SURFACES

6.1 INTRODUCTION

Squeeze films are of practical interest in lubrication theory as they are representative of unsteady situations in hydrodynamic lubrication. The experiments conducted by Needs [1] and Fuks [2,3] to study the phenomenon of squeezing demonstrated a residual film after squeezing. Various explanations are offered for the creation of residual films. For example, Hayward and Isdale [4] suggested that apart from dirt and undissolved particles in the residual film, the surface asperities of the bounding surfaces could be offered as explanations. Derjaguin devised an experimental method to avoid any possible interference of dirt particles and observed an increase in viscosity when traces of polar or surface active substances were present [5,6]. With the development of roughness models, squeeze films between rough surfaces have been studied [7,8]. If certain rheological abnormalities were obsered in thin films, a natural question is that how these rheological anomalies affect the bearing performance with surface asperities. Using micropolar fluid theory, Prakash et.al [9] showed that the rheologicalanomalies like enhancement of viscosity in thin films were rheological effects and could not be attributed to the presence of asperities. In this Chapter,

we study the effects of surface roughness and rheological anomalies such as consistency variation across the film thickness by following a general model for consistency variation discussed in Chapter 5 together with Christensen's theory of roughness in the case of power law lubricants.

6.2 PARALLEL PLATES

Consider the flow of a power law lubricant between two normally approaching infinite parallel plates with a normal velocity V (Fig. 6.1). The plates are of length 2d and are separated by a stochastic film thickness 2h. $2h_1$ is the initial nominal film thickness of the system just prior to squeezing.

(i) ONE DIMENSIONAL LONGITUDINAL ROUGHNESS

The stochastic form of Reynolds equation for this case of roughness can be obtained from eqn. (5.13) and Fig. 6.1 as

$$\frac{d}{dx}\left[\frac{n}{2n+1}\, h_1^{q/n}\, E(f_r)\left(-\frac{1}{m_1}\frac{dE(p)}{dx}\right)^{1/n}\right] = -\frac{dh_n}{dt} \qquad (6.1)$$

where (f_r) is given by

$$(f_r) = \{(1-K^{-1/n})(h_n-a_p)^{(2n+1)/n} + K^{-1/n}\, h^{(2n+1)/n}\} / h^{q/n} \qquad (6.2)$$

where a_p is apparent peripheral layer.
To determine average pressure E(p) we use the following boundary conditions :

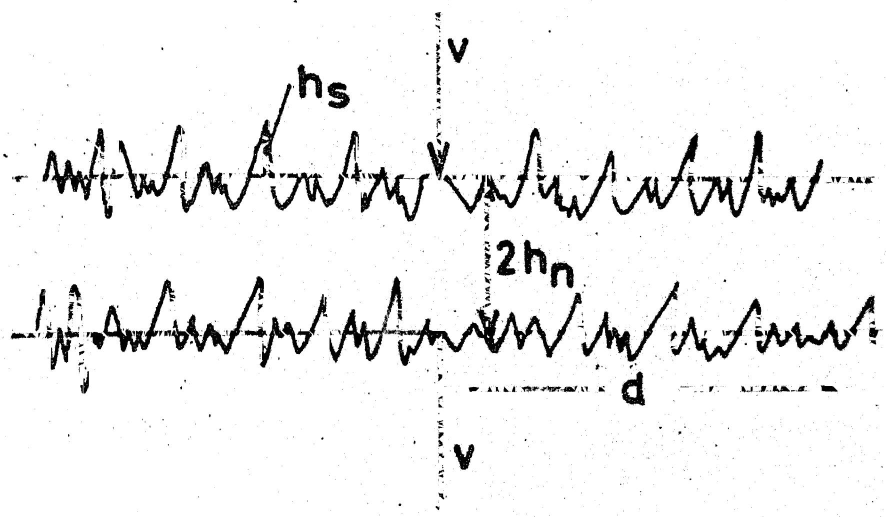

Fig. 6·1 Squeezing between two rough parallel plates

$$\frac{dE(p)}{dx} = 0 \quad \text{at } x = 0$$
$$E(p) = 0 \quad \text{at } x = d \tag{6.3}$$

Integrating eqn. (6.1) and using conditions (6.3), we obtain the average pressure $E(p)$ as

$$E(p) = m_1 \left(-\frac{2n+1}{n} \frac{dh_n}{dt}\right)^n \left(\frac{1}{h_1}\right)^q \left\{\frac{1}{E(f_r)}\right\}^n \frac{d^{n+1} - x^{n+1}}{n+1} \tag{6.4}$$

The load $W_L(n,K,q,b)$ for the case of longitudinal roughness is given by

$$W_L(n,K,q,b) = 2 \int_0^d E(p)\, dx \tag{6.5}$$

Using eqn. (6.4), eqn. (6.5) becomes

$$W_L(n,K,q,b) = \frac{2m_1}{n+2} \left(-\frac{2n+1}{n} \frac{dh_n}{dt}\right)^n \left(\frac{1}{h_1}\right)^q d^{n+2} \left\{\frac{1}{E(f_r)}\right\}^n \tag{6.6}$$

The squeeze time from the initial nominal film thickness $2h_1$ to a subsequent nominal film thickness $2h_2$ is given by

$$t_L(n,K,q,b) = \left(\frac{2m_1}{n+2\; W_L(n,K,q,b)}\right)^{1/n} \left(\frac{2n+1}{n}\right) \left(\frac{1}{h_1}\right)^{q/n} d^{(n+2)/n} \int_{h_2}^{h_1} \frac{dh_n}{E(f_r)} \tag{6.7}$$

The load components $W_S(n,K,q)$ and response time $t_S(n,K,q)$ for the smooth case are given by (eqns. (3.16) and (3.17))

$$W_S(n,K,q) = \frac{2m_1}{n+2} \left(-\frac{2n+1}{n} \frac{dh_n}{dt}\right)^n \left(\frac{1}{h_1}\right)^q d^{n+2} \frac{1}{(f_o)^n\, h_n^{2n+1-q}} \tag{6.8}$$

and

$$t_s(n,K,q) = \left(\frac{2m_1}{(n+2)W_s(n,K,q)}\right)^{1/n} \left(\frac{2n+1}{n}\right)\left(\frac{1}{h_1}\right)^{q/n} d^{(n+2)/n} \int_{h_2}^{h_1} \frac{dh_n}{(f_o)h_n^{(2n+1-q)/n}} \tag{6.9}$$

where (f_o) is given by (eqn. (3.11))

$$(f_o) = 1-(1-K^{-1/n})\{1-(1-a/h_n)^{(2n+1)/n}\} \tag{6.10}$$

h_n being the film thickness in the smooth case and a, peripheral layer.

To study the effect of longitudinal roughness on load and response time we define the quantities $\bar{W}_{LB}$ and $\bar{t}_{LB}$ as follows :

$$\bar{W}_{LB} = \frac{W_L(n,K,q,b)-W_s(n,K,q)}{W_s(n,K,q)} = \frac{(F_o)^n H_n^{2n+1-q}}{\{E((F_r))\}^n} - 1 \tag{6.11}$$

$$\bar{t}_{LB} = \frac{t_L(n,K,q,b)-t_s(n,K,q)}{t_s(n,K,q)}$$

$$= \left\{\int_{H_2}^{1} \frac{dH_n}{\{E((F_r))\}^n} \Big/ \int_{H_2}^{1} \frac{dH_n}{(F_o)H_n^{(2n+1-q)/n}}\right\} - 1 \tag{6.12}$$

where

$$\bar{a}_p = \bar{a}$$

$$(F_r) = \{(1-K^{-1/n})(H_n-\bar{a}_p) + K^{-1/n} H^{(2n+1)/n}\} / H^{q/n} \tag{6.13}$$

$$(F_o) = 1-(1-K^{-1/n})\{1-(1-\bar{a}/H_n)^{(2n+1)/n}\} \tag{6.14}$$

$$H = H_n+H_s,\ H_n = h_n/h_1,\ H_s = h_s/h_1,\ H_2 = h_2/h_1, \bar{a}_p = a_p/h_1 \tag{6.15}$$

$$\bar{a} = a/h_1,\ B = b/h_1$$

To study the effect of consistency variation on load and response time, we define the following quantities using eqn. (6.15):

$$\bar{W}_{LK} = \frac{W_L(n,K,q,b)}{\bar{W}_L(n,1,q,b)} = \frac{[E\{H^{(2n+1-q)/n}\}]^n}{[E((F_r))]^n} \tag{6.16}$$

$$\bar{t}_{LK} = \frac{t_L(n,K,q,b)}{t_L(n,1,q,b)} = \int_{H_2}^{1} \frac{dH_n}{E(F_r)} \Big/ \int_{H_2}^{1} \frac{dH_n}{E(H^{(2n+1-q)/n})} \tag{6.17}$$

(ii) ONE DIMENSIONAL TRANSVERSE ROUGHNESS

In this case, the Reynolds equation is given by (using eqn. (5.50) and Fig.6.1)

$$\frac{d}{dx}\left[\frac{n}{2n+1} h_1^{q/n} \{E(\frac{1}{(f_r)^n})\}^{-1/n} \{-\frac{1}{m_1}\frac{dE(p)}{dx}\}^{1/n}\right] = -\frac{dh_n}{dt} \tag{6.18}$$

Following a procedure similar to that adopted in the case of longitudinal roughness, we obtain the load and response time as

$$W_T(n,K,q,b) = \frac{2m_1}{n+2}\left(-\frac{2n+1}{n}\frac{dh_n}{dt}\right)^n \left(\frac{1}{h_1}\right)^q d^{n+2}\, E\{\frac{1}{(f_r)^n}\} \tag{6.19}$$

and

$$t_T(n,K,q,b) = \left(\frac{2m_1}{n+2\; W_T(n,K,q,b)}\right)^{1/n}\left(\frac{2n+1}{n}\right)\left(\frac{1}{h_1}\right)^{q/n} d^{(n+2)/n} \int_{h_2}^{h_1} E\left(\frac{1}{(f_r)^n}\right)^{1/n} dh_n \tag{6.20}$$

To study the effects of transverse roughness and consistency variation on load and response time, we define the following quantities

$$\bar{W}_{TB} = (F_o)^n H_n^{2n+1-q}\, E\left(\frac{1}{(F_r)^n}\right)^{-1} \tag{6.21}$$

$$\bar{t}_{TB} = \left[\int_{H_2}^{1}\{E(\frac{1}{(F_r)^n})\}^{1/n} dH_n \Big/ \int_{H_2}^{1} \frac{dH_n}{(F_o)H_n^{(2n+1-q)/n}}\right]^{-1} \tag{6.22}$$

$$\bar{W}_{TK} = \frac{E(1/(F_r)^n)}{E(1/H^{2n+1-q})} \tag{6.23}$$

$$\bar{\tau}_{TK} = \int_{H_2}^{1} \{ E(1/(F_r)^n \}^{1/n} dH_n / \int_{H_2}^{1} \{E(1/H^{2n+1-q})\}^{1/n} dH_n \tag{6.24}$$

6.3 CIRCULAR PARALLEL PLATES

The configuration of the system is as shown in Fig. 6.2. The plates are of radius R and separated by a film thickness 2h.

(i) ONE DIMENSIONAL RADIAL ROUGHNESS

The average Reynolds equation in this case can be obtained by using

$$\frac{1}{r}\frac{d}{dr}\left[\frac{n}{2n+1} h_1^{q/n} E((f_r)) r \left(-\frac{1}{m_1}\frac{dE(p)}{dr}\right)^{1/n}\right] = -\frac{dh_n}{dt} \tag{6.25}$$

The boundary conditions are given by

$$\frac{dE(p)}{dr} = 0 \qquad \text{at } r = 0 \tag{6.26}$$

$$E(p) = 0 \qquad \text{at } r = R$$

Integrating eqn. (6.25) using conditions given in eqn. (6.26) we obtain the average pressure E(p) as

$$E(p) = m_1\left(-\frac{2n+1}{2n}\frac{dh_n}{dt}\right)^n \left[\frac{1}{E((f_r))}\right]^n \frac{R^{n+1}-r^{n+1}}{n+1}. \tag{6.27}$$

The load $W_L(n,K,q,b)$ for the case of longitudinal roughness is given by

$$W_L(n,K,q,b) = \int_0^R 2\pi r\, E(p)dr \tag{6.28}$$

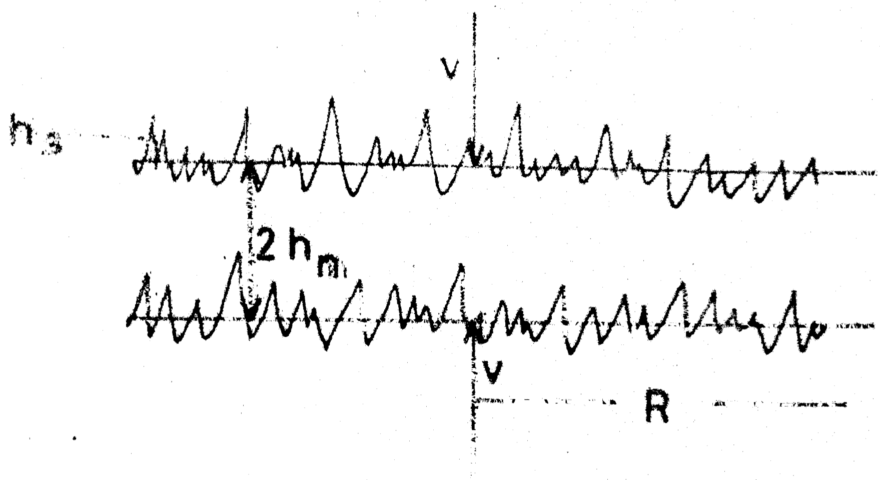

Fig. 6·2 Squeezing between two rough circular plates.

Using eqn. (6.27) in eqn. (6.28) we obtain

$$W_L(n,K,q,b) = \frac{\pi m_1}{n+3}\left(-\frac{2n+1}{2n}\frac{dh_n}{dt}\right)^n\left(\frac{1}{h_1}\right)^q R^{n+3}\left\{\frac{1}{E((f_r))}\right\}^n \quad (6.29)$$

The response time from an initial nominal film thickness $2h_1$ to a subsequent nominal film thickness $2h_2$ is given by

$$t_L(n,K,q,b) = \frac{2n+1}{2n}\left(\frac{\pi m_1 R^{n+3}}{W_L(n,K,q,b)(n+3)}\right)^{1/n}\left(\frac{1}{h_1}\right)^{q/n}\int_{h_2}^{h_1}\frac{dh_n}{E((f_r))} \quad (6.30)$$

The load $W_S(n,K,q)$ and time of squeezing $t_S(n,K,q)$ for smooth case are given by

$$W_S(n,K,q) = \frac{\pi m_1}{n+3}\left(-\frac{2n+1}{2n}\frac{dh_n}{dt}\right)^n\left(\frac{1}{h_1}\right)^q R^{n+3}\frac{1}{(f_o)^n h_n^{2n+1-q}} \quad (6.31)$$

and

$$t_S(n,K,q) = \frac{2n+1}{2n}\left(\frac{\pi m_1 R^{n+3}}{W_L(n,K,q)(n+3)}\right)^{1/n}\left(\frac{1}{h_1}\right)^{q/n}\int_{h_2}^{1}\frac{dh_n}{(f_o)^{-} h_n^{(2n+1-q)/n}} \quad (6.32)$$

To study the effect of longitudinal roughness on load and response time, we define the following quantities :

$$\bar{W}_{LB} = \left[(F_o)^n H_n^{2n+1-q}/\{E((F_r)^n)\}^n\right]-1 \quad (6.33)$$

$$\bar{t}_{LB} = \left[\int_{H_2}^{1}\frac{dH_n}{E((F_r))}\Big/\int_{H_2}^{1}\frac{dH_n}{(F_o)H_n^{(2n+1-q)/n}}\right]-1 \quad (6.34)$$

where forms of function (F_o) and (F_r) and non-dimensional scheme are given by eqns. (6.13)-(6.15).

To study the effect of consistency variation the following

quantities are defined :

$$\bar{W}_{LK} = [E\{H^{(2n+1-q)/n}\} / E(F_r)]^n \tag{6.35}$$

$$\bar{t}_{LK} = \int_{H_2}^{1} \frac{dH_n}{E(F_r)} / \int_{H_2}^{1} \frac{dH_n}{E(H^{(2n+1-q)/n})} \tag{6.36}$$

(ii) ONE DIMENSIONAL CIRCUMFERENCIAL ROUGHNESS

Stochastic form of Reynolds equation in this case can be obtained as

$$\frac{1}{r}\frac{d}{dr}\left[\frac{n}{2n+1} h_1^{q/n} E\left(\frac{1}{(f_r)^n}\right)^{-1/n} r \left\{-\frac{1}{m_1}\frac{dE(p)}{dr}\right\}^{-1/n}\right] = -\frac{dh_n}{dt} \tag{6.37}$$

Integrating eqn. (6.37) using conditions (6.26) we can obtain E(p). As in the case of longitudinal roughness, we can define similar quantities to study the effect of transverse roughness and consistency variation as follows :

$$\bar{W}_{TB} = (F_o)^n H_n^{2n+1-q} E\{1/(F_r)^n\}^{-1} \tag{6.38}$$

$$\bar{t}_{TB} = \left[\int_{H_2}^{1} \{E(1/(F_r)^n\}^{1/n} dH_n / \int_{H_2}^{1} dH_n / ((F_o) H_n^{(2n+1-q)/n})\right]^{-1} \tag{6.39}$$

$$\bar{W}_{TK} = E\{1/(F_r)^n\}/E\{1/H^{2n+1-q}\} \tag{6.40}$$

$$\bar{t}_{TK} = \int_{H_2}^{1} \{E(1/(F_r)^n)\}^{1/n} dH_n / \int_{H_2}^{1} \{E(1/(H^{2n+1-q}))\}^{1/n} dH_n \tag{6.41}$$

6.4 ROLLER BEARING

Consider squeezing between two rough identical rollers approaching each other with a normal velocity V. The configuration of the system is as shown in Fig. (6.3).

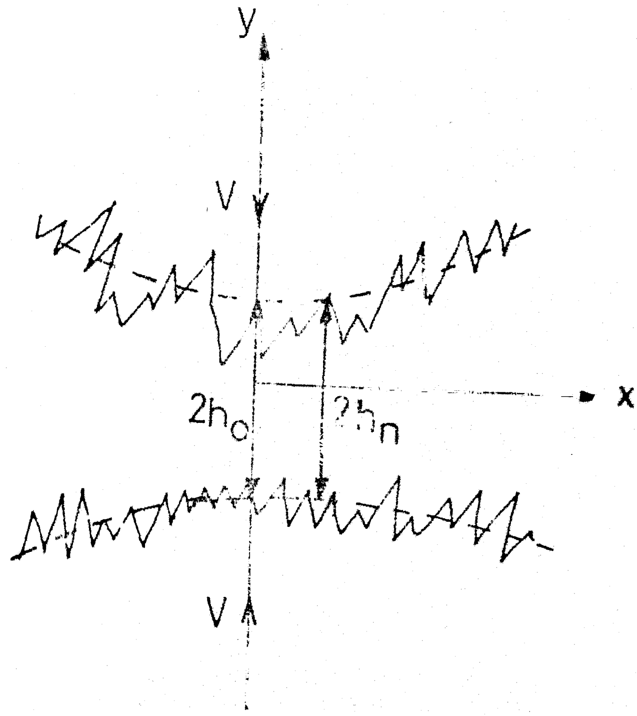

Fig. 6·3 Squeezing between two rough rollers

(i) ONE DIMENSIONAL LONGITUDINAL ROUGHNESS

The average Reynolds equation in this case is obtained from eqn. (5.13) by putting U = 0 as

$$\frac{d}{dx}\left[-\frac{n}{2n+1}\, h_1^{q/n}\, E(f_r)\left(-\frac{1}{m_1}\frac{dE(p)}{dx}\right)^{1/n}\right] = -\frac{\partial h_n}{\partial t} \tag{6.42}$$

where half of nomninal film thickness is given by

$$h_n = h_o + x^2/(2R) \tag{6.43}$$

h_o being nominal minimum film thickness and R the equivalent radius. Using the boundary conditions

$$\frac{dE(p)}{dx} = 0 \quad \text{at } x = 0 \tag{6.44}$$

$$E(p) = 0 \quad \text{at } x = d \tag{6.45}$$

where 2d is the length of the film strectch in the bearing clearance, the average pressure E(p) can be determined as

$$E(p) = -m_1\left(-\frac{2n+1}{n}\frac{\partial h_n}{\partial t}\right)^n \left(\frac{1}{h_1}\right)^q \int_x^d \frac{x^n}{\{E(f_r)\}^n}\, dx \tag{6.46}$$

The load $W_L(n,K,q,b)$ is given by

$$W_L(n,K,q,b) = 2\int_o^d E(p)\,dx \tag{6.47}$$

Using eqn. (6.46), eqn. (6.47) becomes

$$W_L(n,K,q,b) = 2m_1\left(\frac{2n+1}{n}\cdot\frac{dh_n}{dt}\right)^n \left(\frac{1}{h_1}\right)^q \int_o^d \frac{x^{n+1}}{\{E(f_r)\}^n}\, dx \tag{6.48}$$

The squeeze time $t_L(n,K,q,b)$ is given by

$$t_L(n,K,q,b) = \left(\frac{2m_1}{\bar{W}_L(n,K,q,b)}\right)^{1/n}\left(\frac{2n+1}{n}\right)\left(\frac{1}{h_1}\right)^{q/n}\int_{h_2}^{h_1}\left\{\int_0^d \frac{x^{n+1}}{\{E(f_r)\}^n}\,dx\right\}^{1/n} dh_o \tag{6.49}$$

The load ratios $\bar{W}_{LB}$ and $\bar{t}_{LB}$ are given by

$$\bar{W}_{LB} = \left[\int_0^1 \frac{X^{n+1}}{\{E((F_r))\}^n}\,dX \Big/ \int_0^1 \frac{X^{n+1}}{(F_o)^n\, H_n^{2n+1-q}}\,dX\right]^{-1} \tag{6.50}$$

where (F_o) is given by eqn. (6.14), $\bar{a}_p = \bar{a}$ and

$$X = x/d\ ,\ H_n = h_n/h_o = 1+\alpha_o X^2,\ H = h/h_o, H_s = h_s/h_o,$$
$$\alpha_o = d^2/(2Rh_o),\ \bar{a}_p = a_p/h_o,\ \bar{a} = a/h_o \tag{6.51}$$

and

$$\bar{t}_{LB} = \int_{H_2}^1\left\{\int_0^1 \frac{X^{n+1}}{\{E((F_r))\}^n}\,dX\right\}^{1/n} dH_o \Big/ \int_{H_2}^1\left\{\int_0^1 \frac{X^{n+1}}{(F_o)^n\, H_n^{2n+1-q}}\,dX\right\}^{1/n} dH_o \tag{6.52}$$

where

$$X = x/b,\ H = h/h_1 = H_o+\alpha_1 X^2,\ H_o = h_o/h_1, H_s = h_s/h_1,$$
$$\bar{a}_p = a/h_1,\ \bar{a} = a/h_1,\ \alpha_1 = d^2/(2Rh_1) \tag{6.53}$$

The quantities $\bar{W}_{LK}$ and $\bar{t}_{LK}$, for the purpose of studying consistency variation of the lubricant are defined as

$$\bar{W}_{LK} = \int_0^1 \frac{X^{n+1}}{\{(E(F_r))\}^n}\,dX \Big/ \int_0^1 \frac{X^{n+1}}{\{E(H^{(2n+1-q)/n})\}^n}\,dX \tag{6.54}$$

where the expressions (F_r), (F_o) and non-dimensional scheme are given by eqns. (6.13), (6.14) and (6.51) respectively

and

$$\bar{t}_{LK} = \int_{H_2}^{1} \left\{ \int_{0}^{1} \frac{X^{n+1}}{\{E((F_r))\}^n} \right\}^{1/n} dH_o \Big/ \int_{H_2}^{1} \left\{ \int_{0}^{1} \frac{X^{n+1}}{\{E(H^{(2n+1-q)/n})\}^n} \right\}^{1/n} dH_o \tag{6.55}$$

where non-dimensional scheme is given by eqn. (6.53).

(ii) ONE DIMENSIONAL TRANSVERSE ROUGHNESS

The stochastic form of Reynolds equation for this type of roughness is given by

$$\frac{d}{dx}\left[\frac{n}{2n+1} h_1^{q/n} \left\{ E\left(\frac{1}{(f_r)^n}\right) \right\}^{-1/n} \left\{ -\frac{1}{m} \frac{dE(p)}{dx} \right\}^{1/n} \right] = -\frac{\partial h_n}{\partial t} \tag{6.56}$$

The load ratio $\bar{W}_{TB}$ and response time ratio $\bar{t}_{TB}$ are obtained as,

$$\bar{W}_{TB} = \left[\int_{0}^{1} X^{n+1} E\left\{ \frac{1}{(F_r)^n} \right\} dX \Big/ \int_{0}^{1} X^{n+1} \Big/ \{(F_o)^n H_n^{2n+1-q}\} dX \right]^{-1} \tag{6.57}$$

where the expressions (F_r), (F_o) and relevent non-dimensional quantities defined in them are given as in the case of $\bar{W}_{LB}$

and

$$\bar{t}_{TB} = \left[\int_{H_2}^{1} \left\{ \int_{0}^{1} X^{n+1} E\left(\frac{1}{(F_r)^n}\right) dX \right\}^{1/n} dH_o \Big/ \int_{H_2}^{1} \left\{ \int_{0}^{1} \frac{X^{n+1}}{(F_o)^n H_n^{2n+1-q}} dX \right\}^{1/n} dH_o \right]^{-1} \tag{6.58}$$

the expression (F_r), (F_o) and non-dimensional scheme are given as in the case of $\bar{t}_{LB}$.

To study the effect of consistency variation on load and response time we use the following quantities

$$\bar{W}_{TK} = \int_0^1 X^{n+1}\, E\{-\frac{1}{(F_r)^n}\}dX \Big/ \int_0^1 X^{n+1}\, E\{-\frac{1}{H^{2n+1-q}}\}\, dX \qquad (6.59)$$

$$\bar{t}_{TK} = \int_{H_2}^{1} \{\int_0^1 X^{n+1}\, E(\frac{1}{(F_r)^n})dX\}^{1/n}\, dH_o \Big/$$

$$\int_{H_2}^{1} \{\int_0^1 X^{n+1}\, E(\frac{1}{H^{2n+1-q}})dX\}^{1/n}\, dH_o \qquad (6.60)$$

where the function (F_r), (F_o) and non-dimensional scheme for $\bar{W}_{TK}$ and $\bar{t}_{TK}$ are given by those of $\bar{W}_{LK}$ and $\bar{t}_{LK}$ respectively.

6.5 RESULTS AND DISCUSSION

To study the effect of K on load ratio $\bar{W}_{LK}$ and $\bar{W}_{TK}$ in the case of parallel plates, eqns. (6.16) and (6.23) are evaluated for various values of n and depicted together in Fig. 6.4. It is observed that as in the case of squeezing for smooth bearing, an increase in K results in an increase in load capacity for all n for both the cases of longitudinal and transverse roughness ; pseudoplastic lubricants are much more influenced by the consistency variation across the film thickness than the Newtonian and dilatant lubricants. The response time increases both for longitudinal and transverse roughness for increasing K as is evident from Fig. 6.5 (using eqns. 6.55 and 6.60).

Fig. 6.6 gives the picture of response

time vs. roughness parameter B (eqns. 6.52 and 6.58). As B increases, the response time decreases for the longitudinal case and increase for transverse case. Further, the effect of roughness is to increases the response time for increasing flow behaviour index.

The effect of the roughness parameter on load ratio $\bar{W}_{LB}$ and $\bar{W}_{TB}$ is presented in Table 1.

Table 1

Variation of load ratios $\bar{W}_{LB}$ and $\bar{W}_{TB}$ with roughness parameter B

	$\bar{W}_{LB}$			$\bar{W}_{TB}$		
B	m=0.5	n = 1.0	n = 1.5	n = 0.5	n = 1.0	n = 1.5
.1	-0.534	-0.721	-0.832	0.005	0.013	0.024
.2	-0.536	-0.722	-0.833	0.024	0.057	0.107
.3	-0.540	-0.725	-0.835	0.057	0.141	0.274

$K=1.5$, $\bar{a}_p = 0.6$, $q = 0.5$

As the roughness parameter B increases, the load increases in the transverse case and decreases in the longitudinal case for all n. The effect of surface roughness for fixed K on load is more for dilatants compared to Newtonian and pseudoplastic lubricants.

In the case of circular parallel plates, the load and response time ratios obtained for radial and circumferential

one dimensional roughness are the same as those the longitudinal and transverse roughness for parallel plates. Hence, similar interpretations can be accorded to circular parallel plates.

In the case of rollers, eqns. (6.54), (6.55),(6.59) and (6.60) are evaluated to determine the effect of K on load ratio and response time and depicted in Figs. 6.7 and 6.8 respectively. Similar to squeezing for smooth case , the load and response time increase as K increases in the case of rough rollers. These bearing characteristics follow the same trend with regard to flow behaviour index as in the case of smooth rollers under squeezing.

The important aspect studied in this Chapter is the effect of roughness in the lubrication of fluid with layers of different consistencies. The effect of roughness on load ratios $\bar{W}_{LB}$ and $\bar{W}_{TB}$ can be studied by evaluating eqns. (6.50) and (6.57). This is depicted in Fig.6.9. The effect of roughness is to decrease the load capacity for longitudinal case and increase it for transverse case for all n. As n increases, the load ratio increases in the transverse case, implying that the dilatants are capable of supporting more load due to roughness. The dilatant lubricants show an opposite trend with regard to load capacity for the longitudinal case.

Similar behaviour of the dilatants with regard to response time is observed in Fig. 6.10 using eqns. (6.52) and (6.58).

Table 2 gives the picture of the effect of thermal factor q on load ratios $\bar{W}_{LK}$ and $\bar{W}_{TK}$ and response time ratios $\bar{t}_{LK}$ and $\bar{t}_{TK}$. It is observed that for $K > 1$ as q increases the load ratio decreases for the longitudinal roughness whereas it increases for transverse roughness. However, the increase is very small. The response time ratio decreases as q increases for both the cases of roughness structures.

Table 2

Effect of thermal factor q on load ratios $\bar{W}_{LK}$ and $\bar{W}_{TK}$ and response time ratios $\bar{t}_{LK}$ and $\bar{t}_{TK}$ for rollers

q	$\bar{W}_{LK}$	$\bar{W}_{TK}$	$\bar{t}_{LK}$	$\bar{t}_{TK}$
.1	1.3736	1.3645	2.129	2.113
.5	1.3721	1.3651	2.114	2.101
.7	1.3713	1.3653	2.106	2.014
.9	1.3705	1.3656	2.096	2.087

$K = 1.5$, $n = 0.5$, $\bar{a}_p = 0.4$, $B = 0.2$

6.6 CONCLUSION

In the presence of asperities, the effect of consistency variation across the film thickness for squeezing for parallel

infinite plates, circular plates and rollers is the same as that of the smooth case for the respective configurations. The effect of roughness with specified consistency variation parameters is more felt in the case of dilatants compared to Newtonian and pseudoplastic fluids with regard to load capacity and response time. The effect of the thermal factor parameter is to decrease the load ratio for longitudinal roughness and to increase the ratio for transverse roughness when the peripheral layer has greater consistency than that of the central layer. However, the increase due to the effect of q is very small.

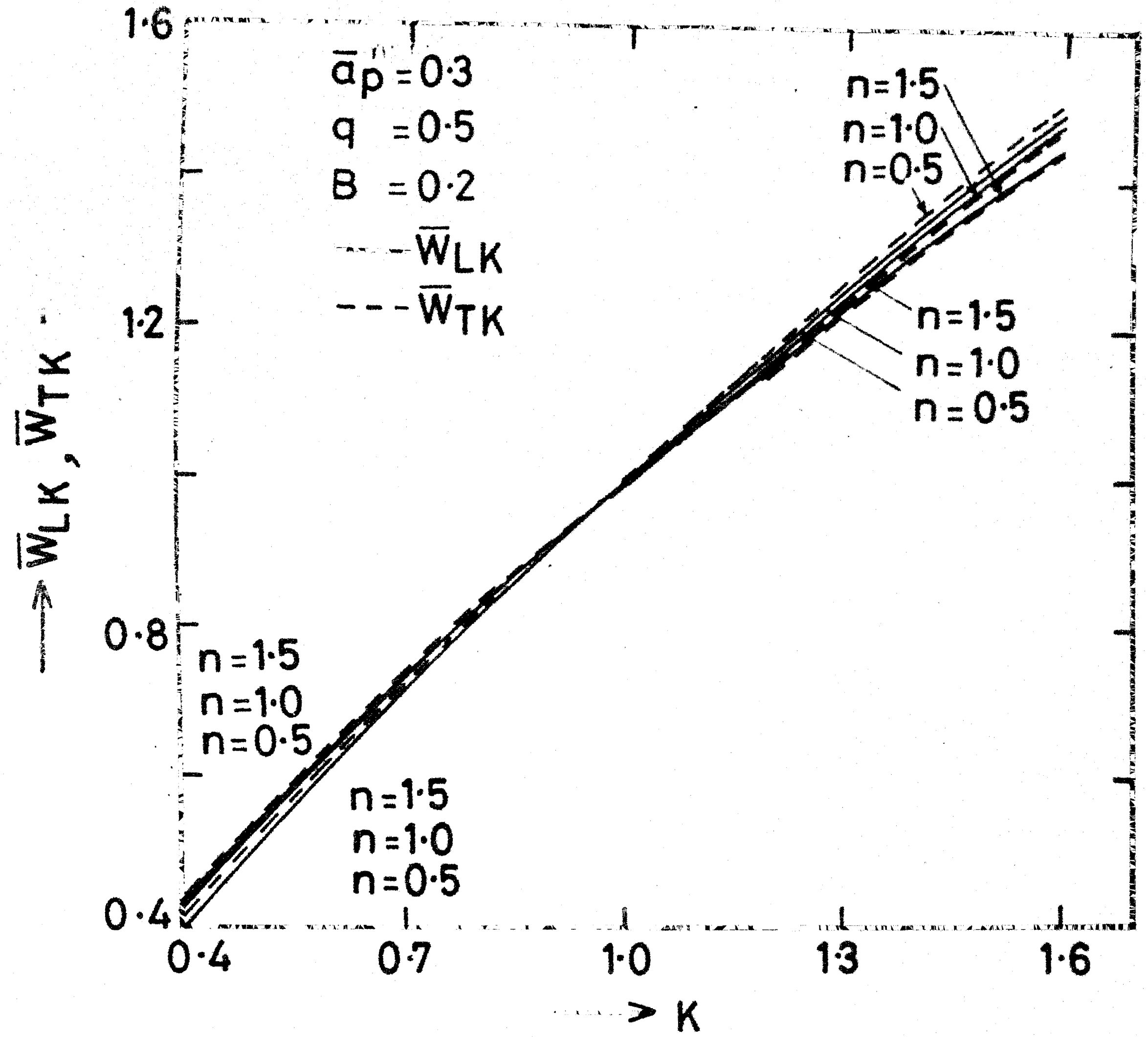

Fig.6 4 The effect of consistency ratio K on load ratios $\bar{W}_{LK}$ and $\bar{W}_{TK}$ for different values of flow behaviour index n for parallel plates

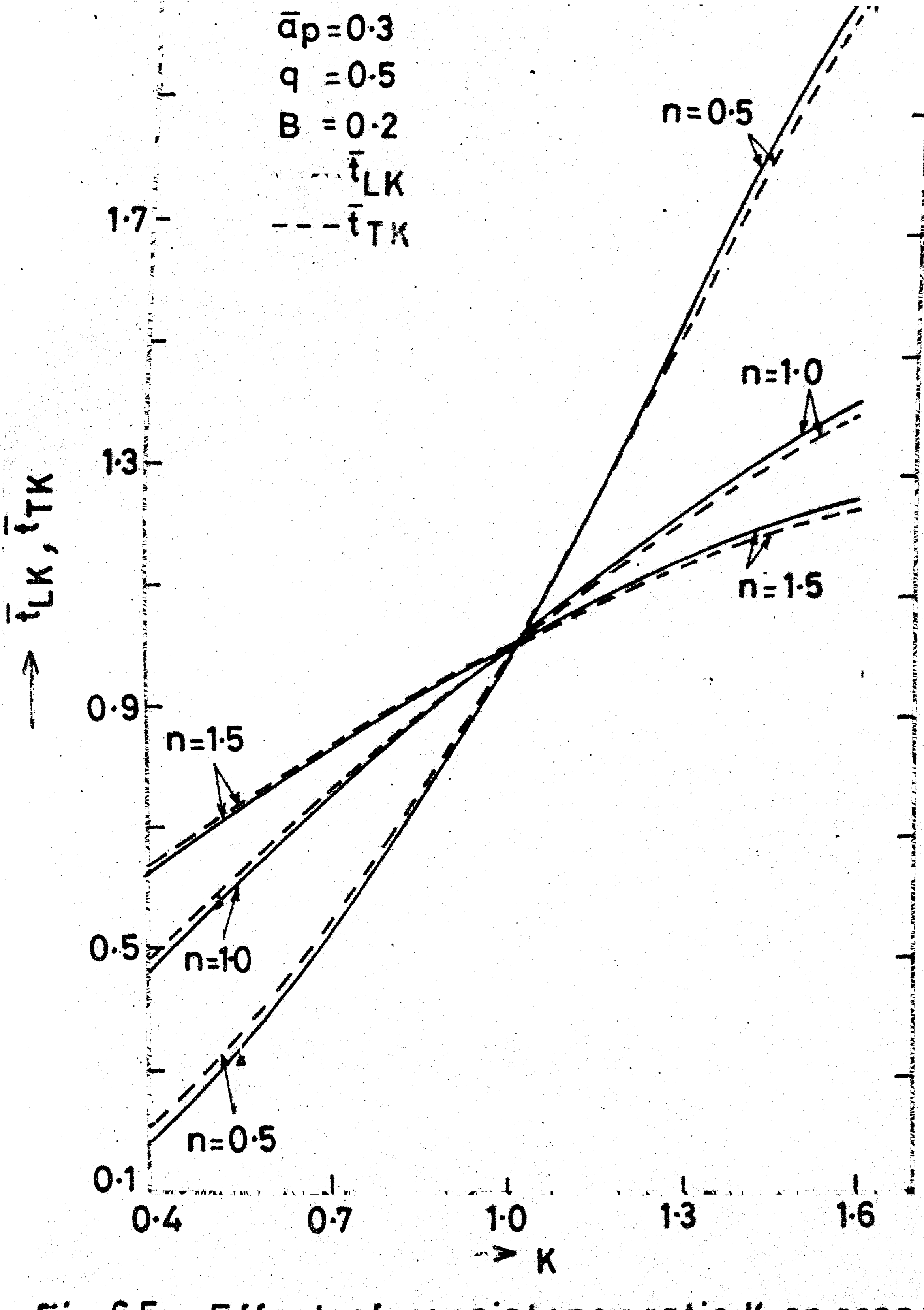

Fig. 6·5 Effect of consistency ratio K on respone time ratios $\bar{t}_{LK}$ and $\bar{t}_{TK}$ for various values of flow behaviour index n for parallel plates

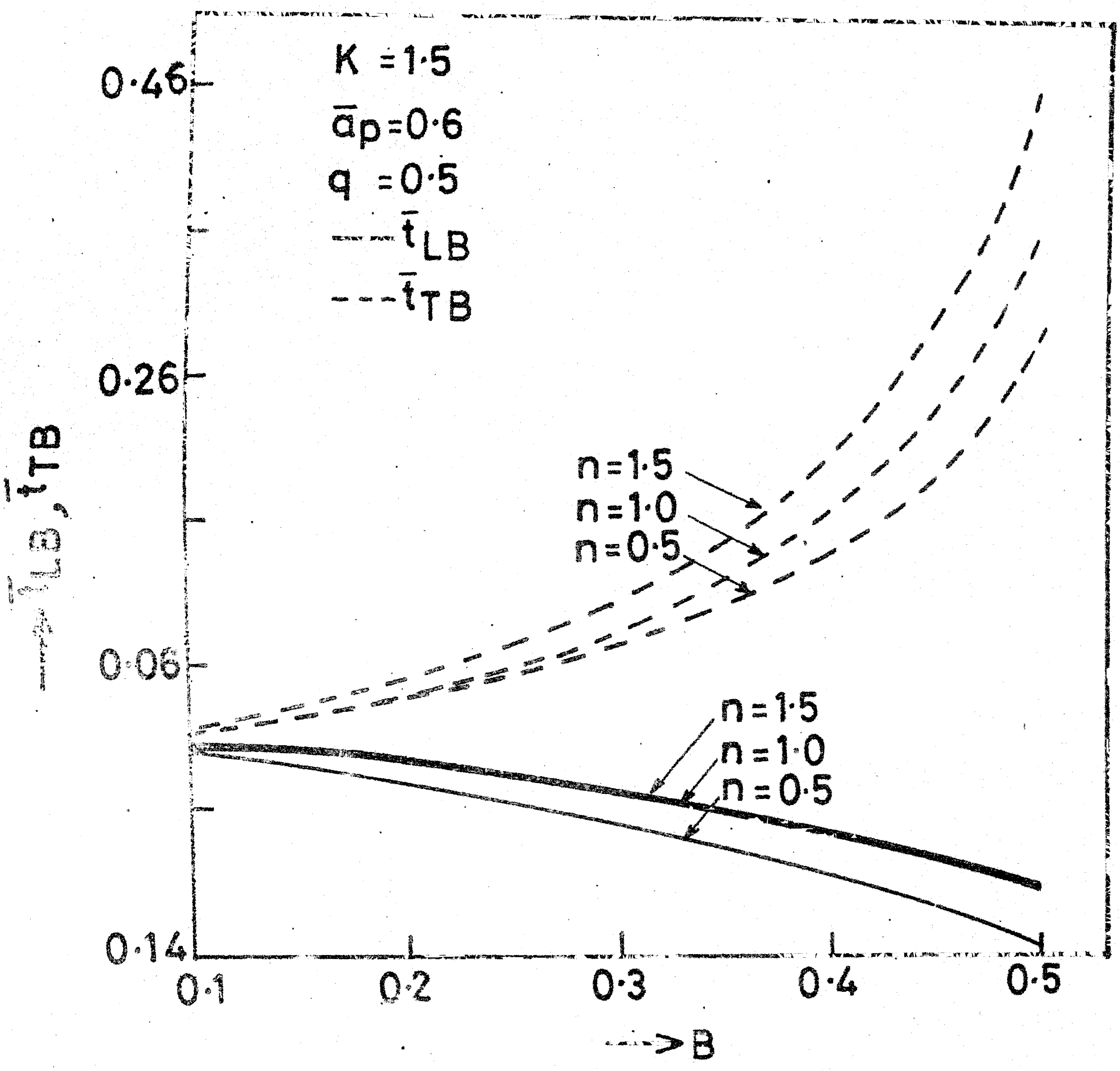

Fig. 6·6 Effect of roughness B on the response time ratio $\bar{t}_{LB}$ and $\bar{t}_{TB}$ for various values of flow behaviour index n for parallel plates

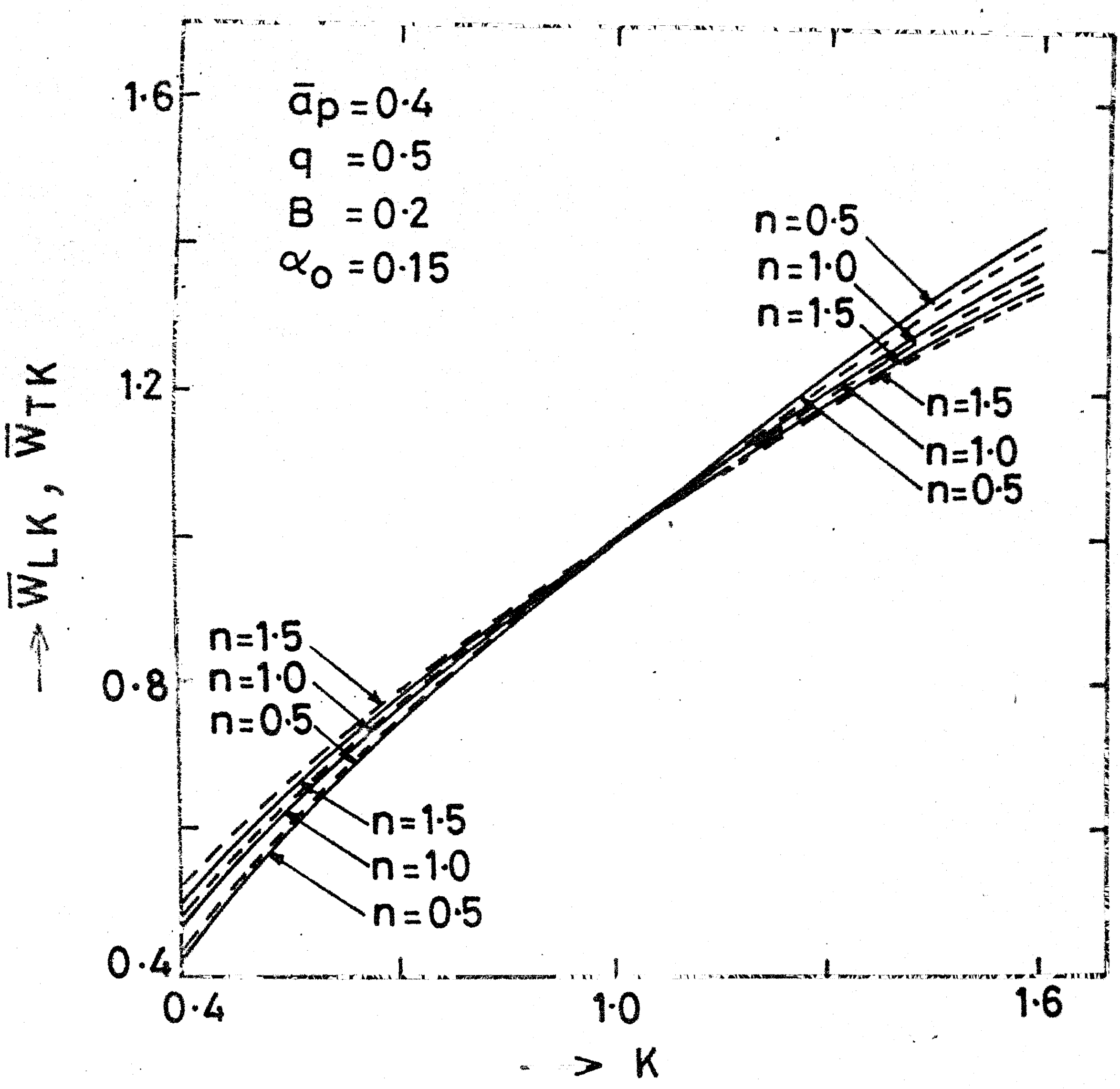

Fig. 6·7 Effect of consistency ratio K on load ratios $\overline{W}_{LK}$ and $\overline{W}_{TK}$ for various values of flow behaviour index n for rollers

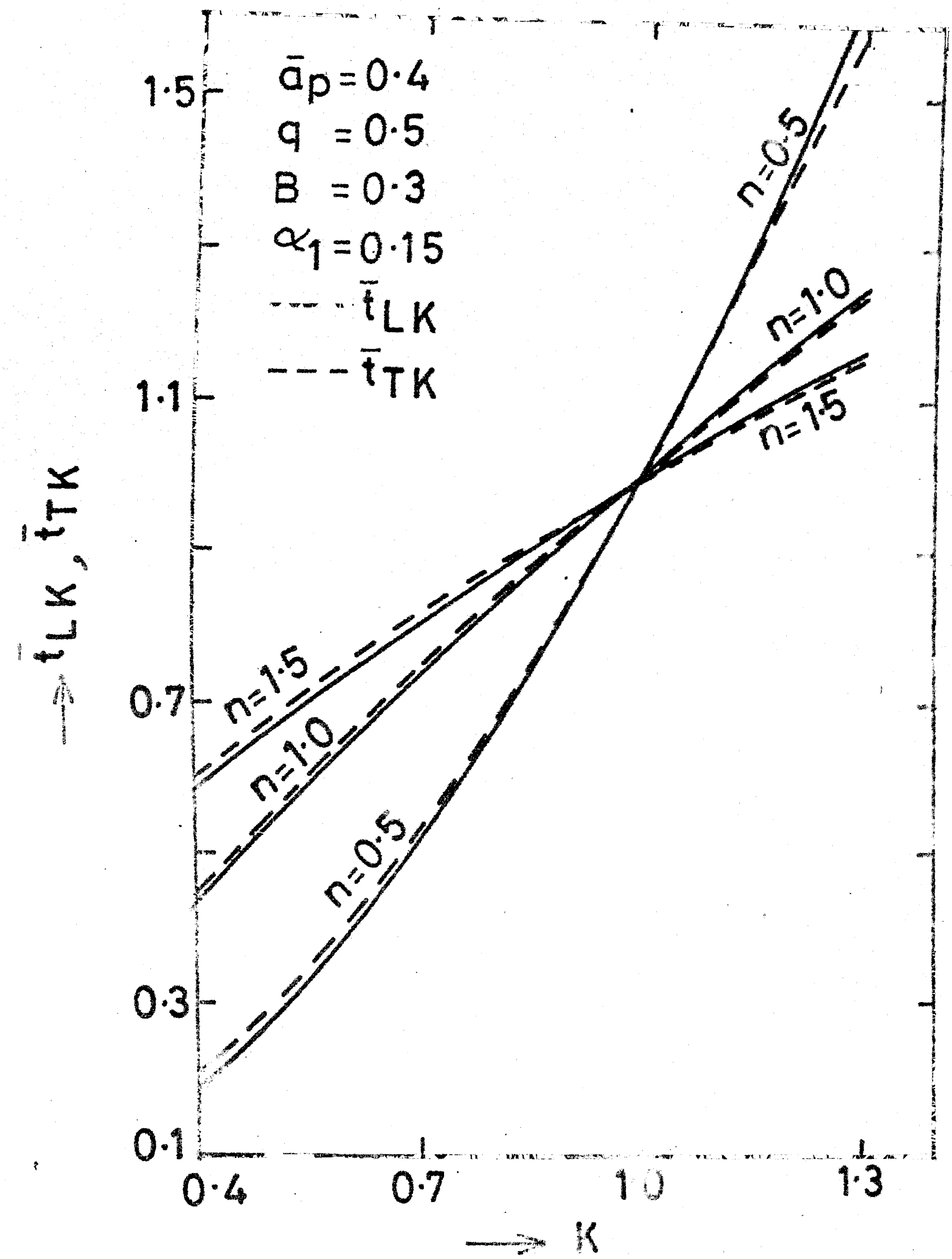

Fig. 6.8 Effect of consistency ratio K on response time ratios $\bar{t}_{LK}$ and $\bar{t}_{TK}$ for various values of flow behaviour index n for rollers

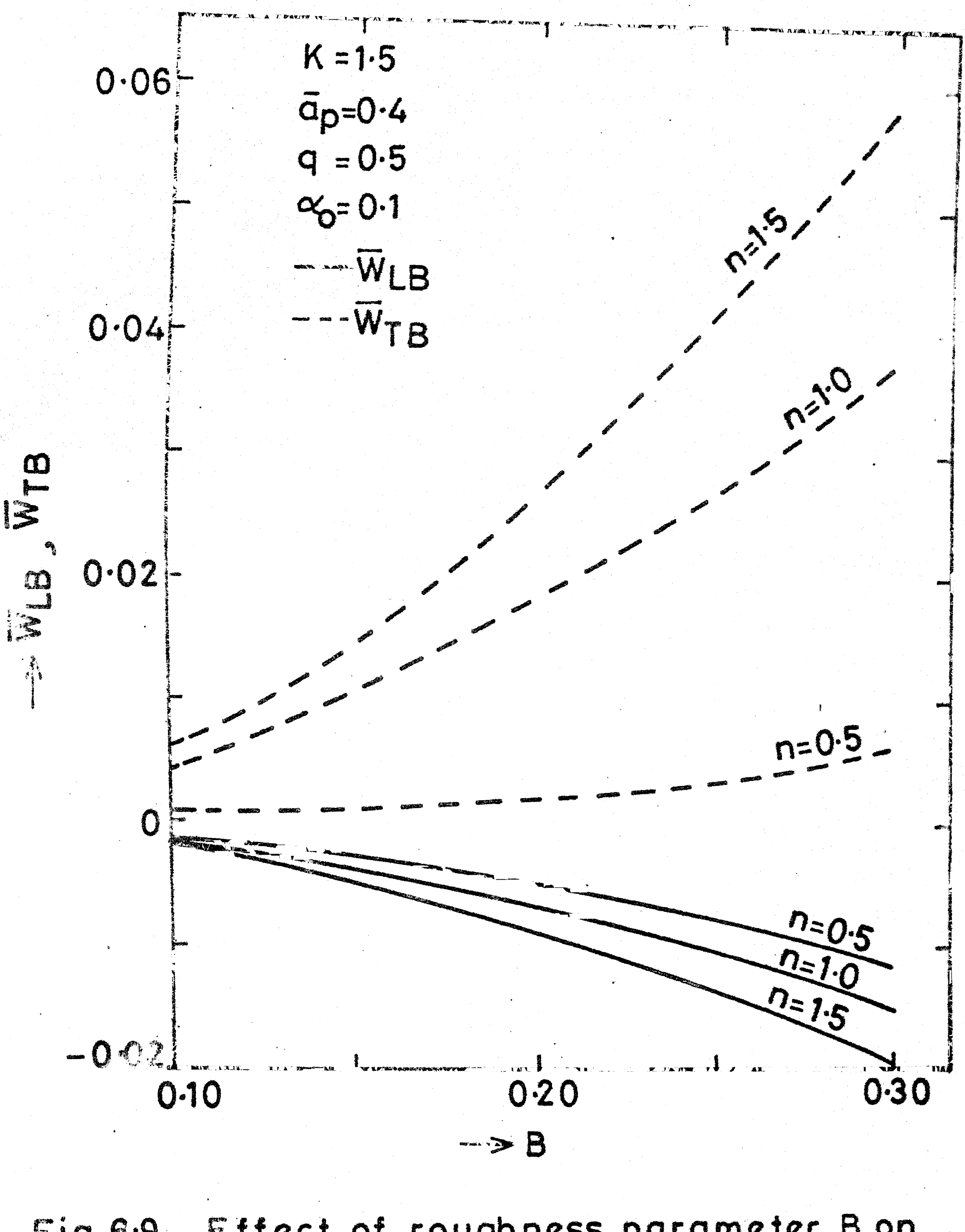

Fig. 6·9 Effect of roughness parameter B on load ratios $\bar{W}_{LB}$ and $\bar{W}_{TB}$ for various values of flow behaviour index for rollers

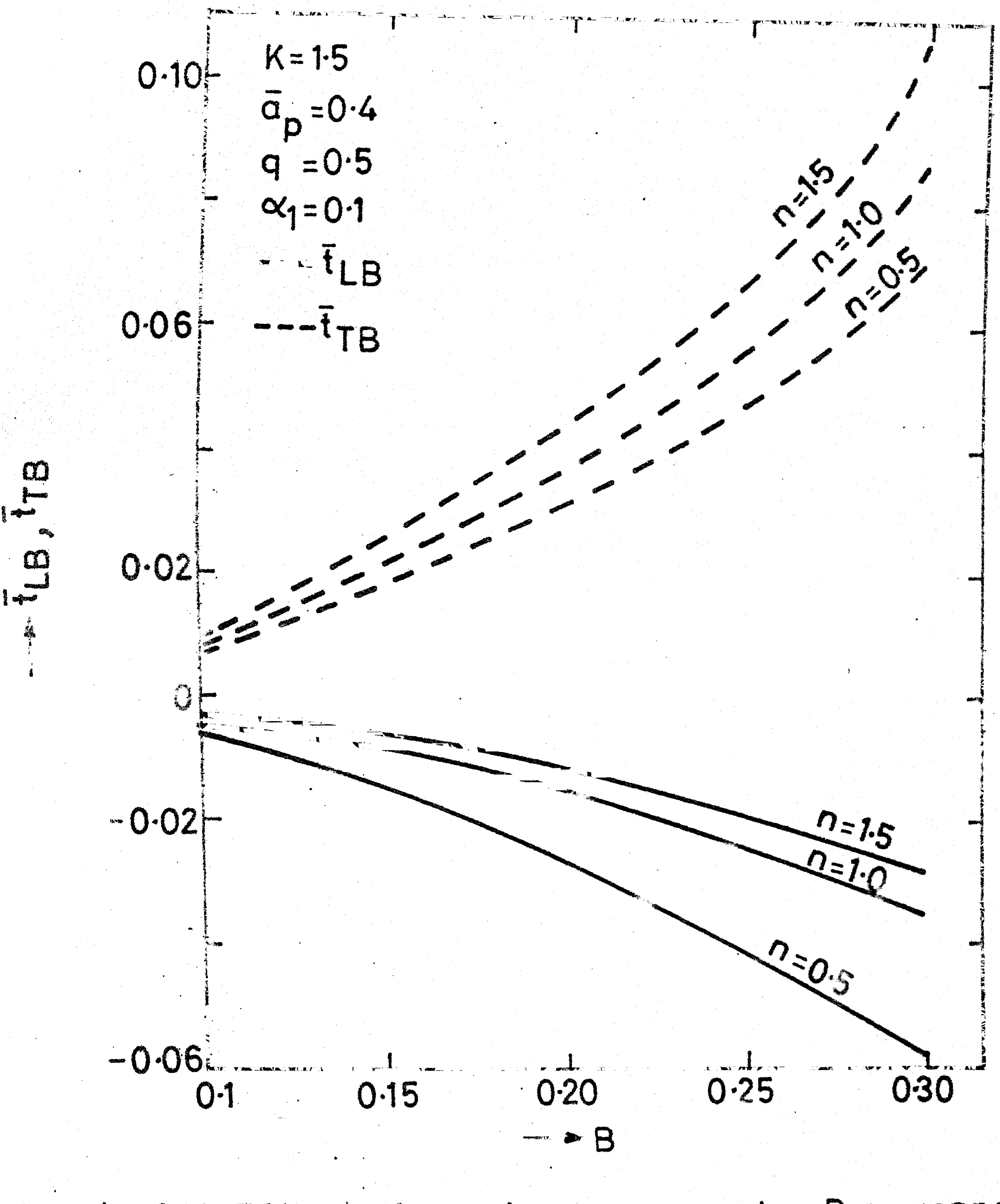

Fig. 6·10 Effect of roughness parameter B on response time ratios $\bar{t}_{LB}$ and $\bar{t}_{TB}$ for various values of flow behaviour index n for rollers

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a_p	apparent peripheral layer film thickness measured from average film thickness
$\bar{a}_p$	non-dimensional quantity corresponding to a_p
b	roughness amplitude measured from average film thickness
$E(\ )$	expectation of ( )
h	stochastic film thickness
h_1, h_2	initial and subsequent film thickness
h_n	nominal film thickness
h_s	roughness part of film thickness
H_1, H_2	non-dimensional quantities corresponding to h_1 and h_2 respectively
H_n, H_s, H_1, H_2, H	non-dimensional quantities corresponding to h_n, h_s, h_1, h_2 and h respectively
d	half the length of rectangular plate or
d	half of film stretch in roller bearing
m, m_1	consistency indices
n	flow behaviour index
p, p_1, p_2	hydrodynamic pressure
q	thermal factor
r	radial distance
R	radius of the circular plate or equivalent radius of rollers
t	time

$\bar{t}_{LB}, \bar{t}_{TB}, \bar{t}_{LK}, \bar{t}_{TK}$ response time ratios defined for longitudinal and transverse cases of roughness and for consistency variation across the film thickness for these cases.

V normal velocity

W_L, W_T, W_S load capacities for longitudinal , transverse roughnesses and smooth case

$\bar{W}_{LB}, \bar{W}_{TB}, \bar{W}_{LK}, \bar{W}_{TK}$ load capacity ratio defined for longitudinal and transverse roughnesses and for consistency variation for these cases respectively

CHAPTER-7

EFFECTS OF CONSISTENCY VARIATION IN ELASTOHYDRODYNAMIC LUBRICATION OF ROUGH ROLLING SURFACES

7.1 INTRODUCTION

Heavily loaded machine components such as gears, cams, rollers etc. operate essentially in elastohydrodynamic regime. The assumption that bearing surfaces are smooth is no longer valid when the bearing operating conditions involve small film thicknesses as in the case of ehd lubrication. Under this condition the surface asperities may interfere and be comparable with the film thickness of the operating lubricant and significantly affect the bearing performance. The topographical structure, deformation mechanism and participation trend of these asperities in lubrication, particularly in the ehd regime constitute one of the major studies in Tribology. Fowles [1] studied the ehd lubrication between identical sliding asperities. Lee and Cheng [2] studied the effect of single asperity on the film and pressure distribution during its entrance into an ehd contact. Tallian [3] studied pressure and traction rippling in ehd contact of rough surfaces.

Attempts were made to include the rheological properties of the lubricant to determine the characteristics of rough bearings. Furey [4] pointed out that lubricant viscosity

and roughness effects had a significant interaction in thin film lubrication. Using micropolar fluid theory, Prakash et.al [5] concluded that the rheological anomalies observed in thin films could not be attributed to the presence of asperities of the bounding surface. Christensen [6] drew attention to the increase in temperature in fluid region possibly by the interaction of asperities in the inlet zone of ehd regime. But, Cheng and Dyson [7] pointed out that in the inlet zone of ehd regime, the isothermal theory could be sufficient for the rollers operating with realistic values of coefficient of friction and upheld the validity of the assumption of equal temperature for bearing surfaces and fluid region. These studies, in general, do not take into consideration the rheological anomalies such as enhanced viscosity in the convicinity of the bearing surface.

In this Chapter, we analyse the elastohydrodynamic lubrication of rollers by considering the surface roughness and consistency variation of the operating power law lubricant. Consistency variation is accounted through the model suggested in Chapter 5. The ehd analysis is limited to the inlet zone as the nominal film thickness, an aspect of primary interest to the design engineer under heavily loaded condition is completely determined by performing such an analysis [8].

7.2 EHD LUBRICATION OF ROUGH ROLLERS

The problem considered is that of the ehd lubrication of rough rollers with a power law lubricant. The consistency of the operating lubricant is considered to vary along as well as across the film thickness. The roughness effects are studied on the pressure distribution in the inlet zone following Christensen's approach [9]. To perform inlet analysis we resort to Grubin's theory [10]. Thus, the shape of the deformed surface at the contact is Hertzian, and in the main load carrying area the surfaces are separated by a parallel fluid film of mean nominal film thickness h_m. It has been pointed out by Jackson and Cameron [11] and Kaneta and Cameron [12] that under pure rolling conditions the surface asperities do not deform in the elastohydrodynamic lubricated (ehl) contact and Hertzian zone is nearly flat. Keeping this in view, in this Chapter, the elastic deformation of the asperities just prior to Hertzian zone is assumed to be negligible [13]. Under typical conditions of ehd lubrication, contacting bodies are much more flexible than either the oil film or surface asperities and deformation of bearing surfaces can be calculated by the Hertzian theory [14]. Recently, Prakash and Czichos [15] pointed out that in the absence of asperity interaction, it is sufficient to analysis the inlet region in the ehl contact.

The geometry of the lubricated rollers deforming under the action of heavy load is depicted in Fig.7.1. In the ehd contact, the reduced pressure q^* and the exponential variation of consistency with pressure are defined as [16]

$$q^* = (1-e^{-\alpha p_E})/\alpha \tag{7.1}$$

$$m_1 = m_o e^{\alpha p_E} \tag{7.2}$$

where α is the pressure-consistency coefficient and m_o is the ambient value of the consistency m_1, appearing in eqn. (2.14). m_1 can be treated to vary with pressure isothermally as the temperature effects can be incorporated in the parameter q in the definition of m in eqn. (2.14). In fact, the general expression for consistency m can be given as [17]

$$m = f(p)(h/h_1)^q \tag{7.3}$$

(i) ONE DIMENSIONAL LONGITUDINAL ROUGHNESS

Consider the deformation of rollers under heavily loaded condition when the peripheral layer thickness a_p is less than half of nominal film thickness h_n. The configuration of deformation of rollers is depicted in Fig. 7.1, where 2d is the width of Hertzian region. Pressure is maximum just outside the parallel width of the lubricant film, i.e. at $x = -d$.

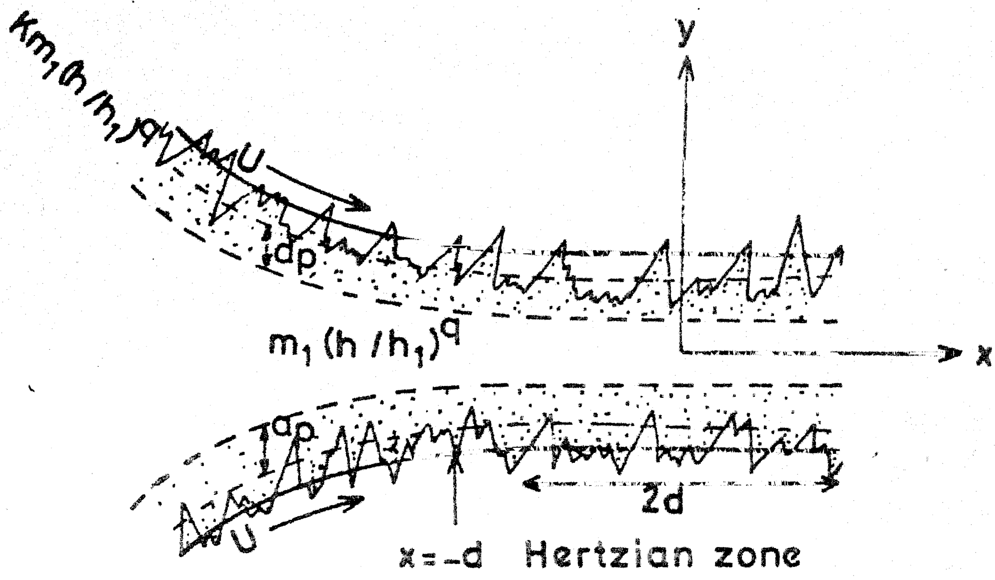

Fig.7·1 Elastohydrodynamic lubrication between two rough rollers considering consistency variation $h_m \gg 2a_p$

With the help of eqns. (7.1), (7.2) and (5.15), the modified Reynolds equation for the case of longitudinal roughness can be written as

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^q \, E(f_r)\left(\frac{1}{m_o}\frac{dE(q^*)}{dx}\right)^{1/n}\right] = U\frac{dh_n}{dx}$$

$$-\infty \leq x \leq -d \qquad (7.4)$$

where h_1 denotes the nominal inlet film thickness which is sufficiently away from the contact zone where q is measured ; d is given by eqn. (2.50)

$$d = 2[W_E \, R_E/E'], \qquad (7.5)$$

where $(\pi/2)/E' = (1-\nu^2)/E_1, \; R_E = r_E/2 \qquad (7.6)$

W_E, R_E, E_1 and ν being load in the ehd lubrication, radius of the equivalent roller, E_1 Young's modulus and ν Poisson's ratio of the rollers.

The expression (f_r) is given by (eqn. 5.10)

$$(f_r) = \{(1-K^{-1/n})(h_n-2a_p)^{(2n+1)/n} + K^{-1/n}\, h^{(2n+1)/n}\} / h^{q/n} \qquad (7.7)$$

The smooth part of the film thickness h_n outside the band of contact width is given by (using eqn. 2.54)

$$h_n = h_m + d^2/(2R)[\,|x/d|\sqrt{(x^2/d^2)-1}-\ln((|x|/d)+\sqrt{(x^2/d^2-1)}] \qquad (7.8)$$

where $h = h_m$ is the minimum nominal film thickness

The boundary conditions to determine pressure $E(q^*)$ are the following :

$$E(q^*) = 0 \text{ at } x = -\infty \qquad (7.9)$$

$$\frac{dE(q^*)}{dx} = 0 \text{ at } x = -d \qquad (7.10)$$

Integrating eqn. (7.4) with conditions (7.9) and (7.10), we obtain average pressure $E(q^*)$ for the inlet zone. Denoting $E(q^*)$ at $x = -d$ for the longitudinal roughness case as q_L^* we have

$$q_L^* = [E(q^*)]_{x=-d}$$

$$= \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_o U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-d} \frac{(h_n-h_m)^n}{\{E(f_r)\}^n} dx \qquad (7.11)$$

The reduced pressure q^* obtained for the smooth case in Chapter 2 (eqn. 2.59) at x = -d, denoted by q_S^* is the given by (eqn. (2.59))

$$q_S^* = [q^*]_{x=-d}$$

$$= \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_o U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-d} \frac{(h_n-h_m)^n}{(f_o)^n h_n^{2n+1-q}} dx \qquad (7.12)$$

where

$$(f_o) = 1-(1-K^{-1/n}) \{1-(1-2a/h_n)^{(2n+1)/n}\} \qquad (7.13)$$

To study the effect of longitudinal roughness on the pressure distribution in the inlet region, we define - a quantity Q_L^* given by

$$Q_L^* = \frac{q_L^*-q_S^*}{q_S} = \left\{ \int_{-\infty}^{-1} \frac{(H_n-H_m)^n}{\{E((F_r))\}^n} dx \Big/ \int_{-\infty}^{-1} \frac{(H_n-H_m)^n}{(F_o)^n H_n^{2n+1-q}} dX \right\} -1 \qquad (7.14)$$

where

$$H = (hE')/(2W_E) = H_n+H_s, \quad H_n = (h_nE')/(2W_E), H_s = (h_sE')/(2W_E) \qquad (7.15)$$

$$H_m = (h_mE')/(2W_E), \quad \bar{a} = (aE')/(2W_E), \quad \bar{a}_p = (a_pE')/(2W_E), \quad B = (bE')/(2W_E)$$

$$H_n = H_m + |X| \sqrt{X^2-1} - \ln(|X| + \sqrt{X^2-1}) \qquad (7.16)$$

$$(F_o) = 1-(1-K^{-1/n})\{(1-2\bar{a}/H_n\}^{(2n+1)/n} \qquad (7.17)$$

$$(F_r) = \{(1-K^{-1/n})(H_n-2\bar{a}_p)^{(2n+1)/n} + K^{-1/n} H^{(2n+1)/n}\} / H^{q/n} \qquad (7.18)$$

The result obtained in eqn. (7.13) for Q_L^* is valid for $h_n \geq 2a_p$. To obtain the factor Q_L^* for $h_n < 2a_p$, we divide the inlet zone into two regions $-\infty \leq x \leq -d_1$ where $h_n \geq 2a_p$ and

$-d_1 \leq x \leq -d$, where $h_n \leq 2a_p$ and $h_n = 2a_p = h_m$ at $x = -d_1$ (Fig. 7.2).

The governing differential eqn. for reduced pressure $q^* = q_1^*$ in the region $-\infty \leq x \leq -d_1$ can be written from eqn. (7.4) as

$$\frac{d}{dx}\left[-\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} E(f_r)\left(\frac{1}{m_o}\frac{dE(q_1^*)}{dx}\right)^{1/n}\right] = U\frac{dh_n}{dx} \tag{7.19}$$

where q_1^* represents the reduced pressure q^* for $h_n \geq 2a_p$.
In the region $-d_1 \leq x \leq -d$, the consistency of the film is $Km_1(h/h_1)^q$ over the entire region and we can write the differential eqn. for pressure q_2 in this region as

$$\frac{d}{dx}\left[-\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n} E(h^{2n+1-q)/n})\left(\frac{1}{Km_n}\frac{dE(q_2^*)}{dx}\right)^{1/n}\right] = U\frac{dh_n}{dx} \tag{7.20}$$

where q_2 denotes the reduced pressure q^* for $h_n \leq 2a_p$.
To determine reduced pressure from eqns. (7.18) and (7.19) we use the following conditions

$$E(q_1^*) = 0 \quad \text{at} \quad x = -\infty \tag{7.21}$$

$$\frac{dE(q_1^*)}{dx} = \frac{dE(q_2^*)}{dx} \quad \text{at } x = -d_1 \tag{7.22}$$

$$\frac{dE(q_2^*)}{dx} = 0 \quad \text{at } x = -d,\ h_n = h_m \tag{7.23}$$

Integrating eqn. (7.19) we get

$$\frac{dE(q_1^*)}{dx} = \left(\frac{2n+1}{2n}\right)^n m_o U^n \left(\frac{1}{h_1}\right)^q \frac{(hn+h_L)^n}{\{E(f_r)\}^n} \tag{7.24}$$

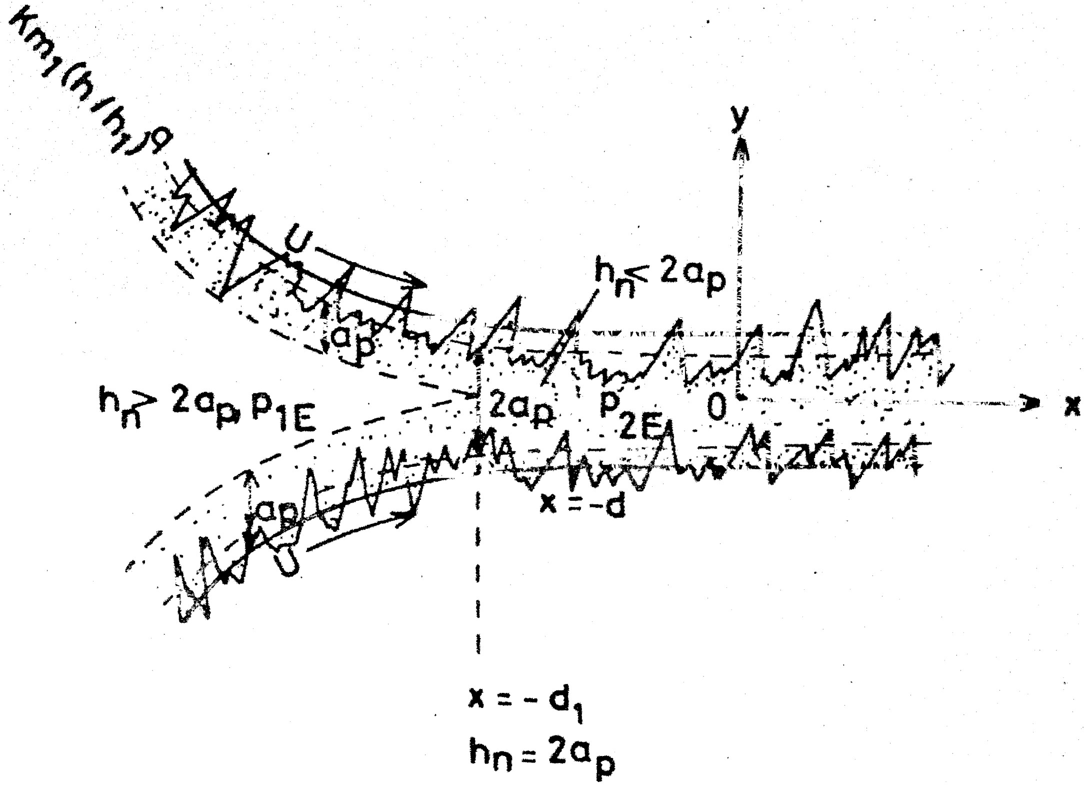

Fig.7·2 Elastohydrodynmic lubrication between two rough rollers considering consistency variation $h_m \leq 2a_p$

where h_L is the constant of integration to be determined subject to the condition $h_n = 2a$ at $x = -d$.

Integrating eqn. (7.19) subject to condition (7.23) we have

$$\frac{dE(q_2^*)}{dx} = \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} U^n K m_o \left(\frac{1}{h_1}\right)^q \frac{(h_n-h_m)^n}{\{E(h^{(2n+1-q)/n})\}^n} \quad (7.25)$$

where $hn = h_m$ at $x = -d$.

Using condition (7.21) in eqns. (7.18) and (7.19), h_L can be determined as

$$h_L = K^{1/n}(2a_p-h_m)\left[\frac{E((f_r))}{E(h^{(2n+1-q)/n})}\right]_{h_n=2a_p} -2a_p \quad (7.26)$$

As in the case of $h_n \geq 2a_p$, we obtain the reduced pressure $E(q_2^*)$ at $x = -d$ as

$$q_L^* = [E(q_2^*)]_{x=-d}$$

$$=\left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_o U^n \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-d_1} \frac{(h_n+h_L)^n}{\{E((f_r))\}^n} dx + K\int_{-d_1}^{d} \frac{(h_n-h_m)^n}{\{E(h^{(2n+1-q)/n})\}^n} dx\right.$$

$$(7.27)$$

Using the dimensionless quantities given in eqn. 7.15 we define the pressure ratio factor Q_L^* for $Hn \leq 2\bar{a}_p$ as

$$Q_L^* = \left\{\int_{-\infty}^{-D_1} \frac{(H_n+H_L)^n}{\{E((F_r))\}^n} dX + K\int_{-D_1}^{1} \frac{(H_n-H_m)^m}{\{E((F_r))\}^n} dX\right\} \Big/ \left\{\int_{-\infty}^{D_1} \frac{(H_n-H_m)^n}{(F_o)^n H_n^{2n+1-q}} dX\right.$$

$$\left. + K\int_{-D_1}^{1} \frac{(H_n-H_m)^n}{H_n^{2n+1-q}} dX \right\}] \quad (7.28)$$

where $D_1 = d_1/d$ and

$$H_L = h_L/h_m = K^{1/n}(2\bar{a}_p - H_n)\left[\frac{E((F_r))}{E(H^{(2n+1-q)/n})}\right]_{H_n=2\bar{a}_p}^{-2\bar{a}_p} \tag{7.29}$$

(ii) ONE DIMENSIONAL TRANSVERSE ROUGHNESS

For this type of roughness, the differential equation to determine reduced pressure for the case $h_n \geq 2a$ can be rewritten from eqn. (5.49) as

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\left\{E\left(\frac{1}{(f_r)^n}\right)\right\}^{-1/n}\left(\frac{1}{m_o}\frac{dE(q^*)}{dx}\right)^{1/n}\right]$$

$$= U\frac{dh_n}{dx} \quad -\infty \leq x \leq -d \tag{7.30}$$

Using the boundary conditions (7.21)-(7.23) and following a similar procedure adopted in the case of longitudinal roughness, we obtain the reduced pressure q_T^* at $x = -d$ as

$$q_T^* = [E(q^*)]_{x=-d}$$

$$= \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_o U^n \left(\frac{1}{h_1}\right)^q \int_{-\infty}^{-d} (h_n - h_m)^n E\left(\frac{1}{(f_r)^n}\right) dx \tag{7.31}$$

and pressure ratio factor Q_T as

$$Q_T^* = \left[\int_{-\infty}^{-1} (H_n - H_m)^n E\left(\frac{1}{(F_r)^n}\right) dX \Big/ \int_{-\infty}^{-1} \frac{(H_n - H_m)^n}{(F_o)^n H_n^{2n+1-q}} dX\right]^{-1} \tag{7.32}$$

In the case of $h_n \leq 2a$, as in the longitudinal case, the inlet region is divided into two region where $h_n \geq 2a$ and $h_n \leq 2a$ and the set of equations to determine reduced pressure in these regions can be obtained as

$$\frac{d}{dx}\left[\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\left\{E\left(\frac{1}{(f_r)^n}\right)\right\}^{-1/n}\left(\frac{1}{m_o}\frac{dE(q_1^*)}{dx}\right)^{1/n}\right]$$

$$= U\frac{dh_n}{dx} \qquad -\infty \leq x \leq -d_1 \qquad (7.33')$$

$$\frac{d}{dx}\left[-\frac{2n}{2n+1}\left(\frac{1}{2}\right)^{(2n+1)/n} h_1^{q/n}\left\{E\left(\frac{1}{h^{2n+1-q}}\right)\right\}^{-1/n}\left(\frac{1}{Km_o}\frac{dE(q_2)}{dx}\right)^{1/n}\right]$$

$$= U\frac{dh_n}{dx} \qquad -d_1 \leq x \leq d \qquad (7.34)$$

where q_1^* and q_2^* represent reduced pressure q^* for the regions defined in eqns. (7.33) and (7.34) respectively. Using the boundary conditions (7.21)-(7.23) we can obtain the reduced pressure q_T^* at $x = -d$ as

$$q_T^* = [E(q_2^*)]_{x=-d}$$

$$= \left(\frac{2n+1}{2n}\right)^n 2^{2n+1} m_o U^n \left(\frac{1}{h_1}\right)^q \left[\int_{-\infty}^{-d_1} (h_n+h_T)^n E\left(\frac{1}{(f_r)^n}\right) dx\right.$$

$$\left. + K\int_{-d_1}^{d} (h_n-h_m)^n E\left(\frac{1}{(f_r)^n}\right) dx\right] \qquad (7.35)$$

where

$$h_T = K^{1/n}(2a_p-h_m)\left[\frac{E\left(\frac{1}{h^{2n+1-q}}\right)}{E\left(\frac{1}{(f_r)^n}\right)}\right]^{1/n}_{h_n=2a_p} -2a_p \qquad (7.36)$$

and pressure ratio factor Q^* as

$$Q_T^* = \{ \int_{-\infty}^{-D_1} (H_n+H_T)^n E(\frac{1}{(F_r)^n}) dX$$

$$+K \int_{-D_1}^{1} (H_n-H_m)^n E(\frac{1}{H^{2n+1-q}}) dX] \} / \{ \int_{-\infty}^{-D_1} \frac{(H_n-H_m)^n}{(F_o)^n H^{2n+1-q}} +$$

$$K \int_{-D_1}^{1} \frac{(H_n-H_m)^n}{H^{2n+1-q}} dX \} -1 \qquad (7.37)$$

H_T is given by

$$H_T = K^{1/n}(2\bar{a}_p - H_m)\left[\frac{E(1/H^{2n+1-q})}{E(1/((F_r)^n)} \right]^{1/n}_{H_n = 2\bar{a}_p} - 2\bar{a}_p \qquad (7.38)$$

7.3 RESULTS AND DISCUSSION

The pressure ratio factors Q_L^* and Q_T^* can be evaluated from eqns. (7.14) and (7.32) respectively in the case of $h_m \geq 2a_p$ for various values of roughness parameter B, the consistency ratio K and thermal factor q. Fig. 7.3 represents the variation of Q_L^* and Q_T^* with B for various values of n. It is observed that as B increases, Q_T^* increases ; the increase is enhanced as flow behaviour index increases. With increasing value of B, Q_L^* decreases and the decrease is ehanced with increasing n. However, the decrease is not significant. The results are in good qualitative agreement with those given in the reference [30].

Fig. 7.4 gives the picture of the variation of Q_L^* and Q_T^* with K in the presence of asperities. For very high and low

values of K, the dilatants are much more influenced than Newtonian and pseudoplastic lubricants. The influence results in increasing inlet pressure for K > 1 and this pressure decreases for K < 1 for dilatants. However, the variation is small, indicating thereby the rheological effects cannot be attributed to the presence of surface asperities. This is in conformity with the results of Prakash et. al [5] who studied slider bearing using a micropolar fluid.

When $h_m \leq 2a_p$, the pressure ratio factors Q_L^* and Q_T^* are evaluated by using eqns. (7.28) and (7.37) and depicted in Figs. 7.5 and 7.6. In the pressence of surface asperities, the effect of K is to increase the pressure ratio factor for both longitudinal and transverse cases for all K (Fig.7.5). For K > 1, the increase of this factor due to the effect of K is more for dilatants for both the roughness patterns. For K < 1, the trend is reversed. The effect of the roughness parameter B is to increase the pressure ratio factor for transverse roughness and decrease it for longitudinal roughness (Fig.7.6) .

These results may be interpreted in terms of nominal film thickness at the inlet. An increase or decrease of pressure ratio factor will result in an increase or decrease in the nominal inlet film thickness. Thus, it can be concluded that the transverse roughness increases the nominal inlet film thickness and the longitudinal roughness decreases it. Larger the value of flow behaviour index n, greater the rate of increase or decrease of nominal inlet film thickness.

7.4 CONCLUSION

In the ehd lubrication of rough rollers, an increase in the consistency in the peripheral layer results in an increase in the pressure ratio factor or inlet film thickness. In case of transverse roughness, the pressure ratio factor increases as the value of the roughness parameter increases, for both the cases when the total peripheral layer thickness on the two rollers is greater or less than the minimum nominal film thickness. In the presence of consistency variation across the film thickness it can be concluded that the dilatants are more effective than pseudoplastics in the case of transverse roughness as compared to the case of longitudinal roughness. The effect of the thermal factor is to the decrease the pressure ratio factor for transverse case and to increase it for longitudinal case. It is concluded that the rheological anomalies in the lubrication process cannot be attributed to the presence of surface asperities for non-Newtonian lubricants characterized by the power law model.

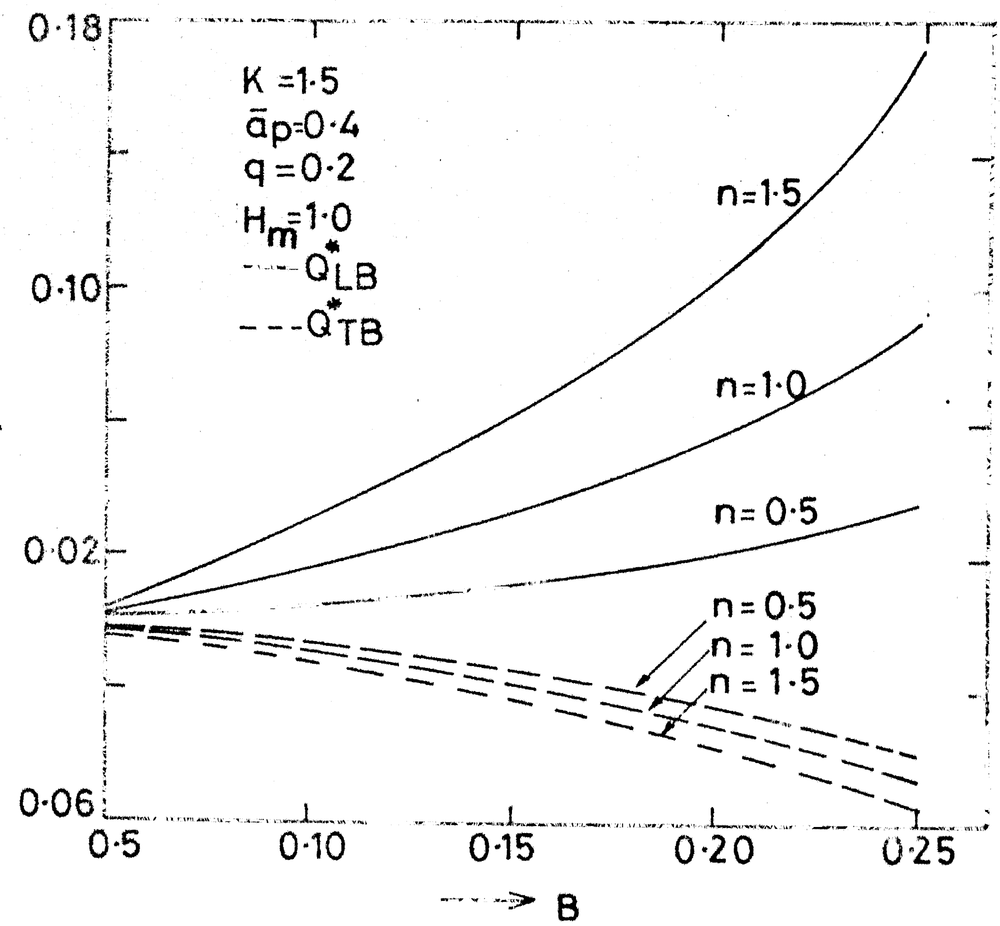

Fig. 7·3 Effect of roughness parameter B on pressure ratio factors Q_L^* and Q_T^* for various values of flow behaviour index n, $H_m \geqslant 2\bar{a}_p$.

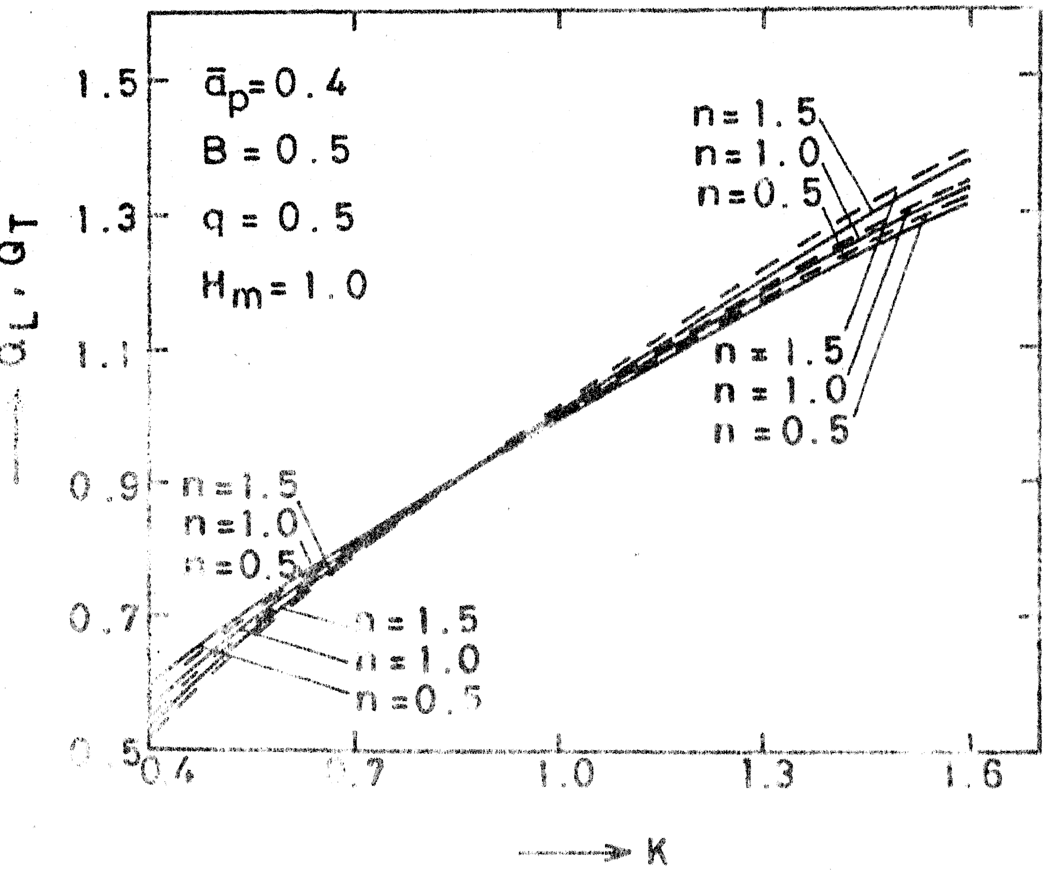

Fig 7.4 Effect of consistency ratio K on pressure ratio facto Q_L^* and Q_T^* for various values of flow behaviour i n, $H_m \geq 2\bar{a}_p$

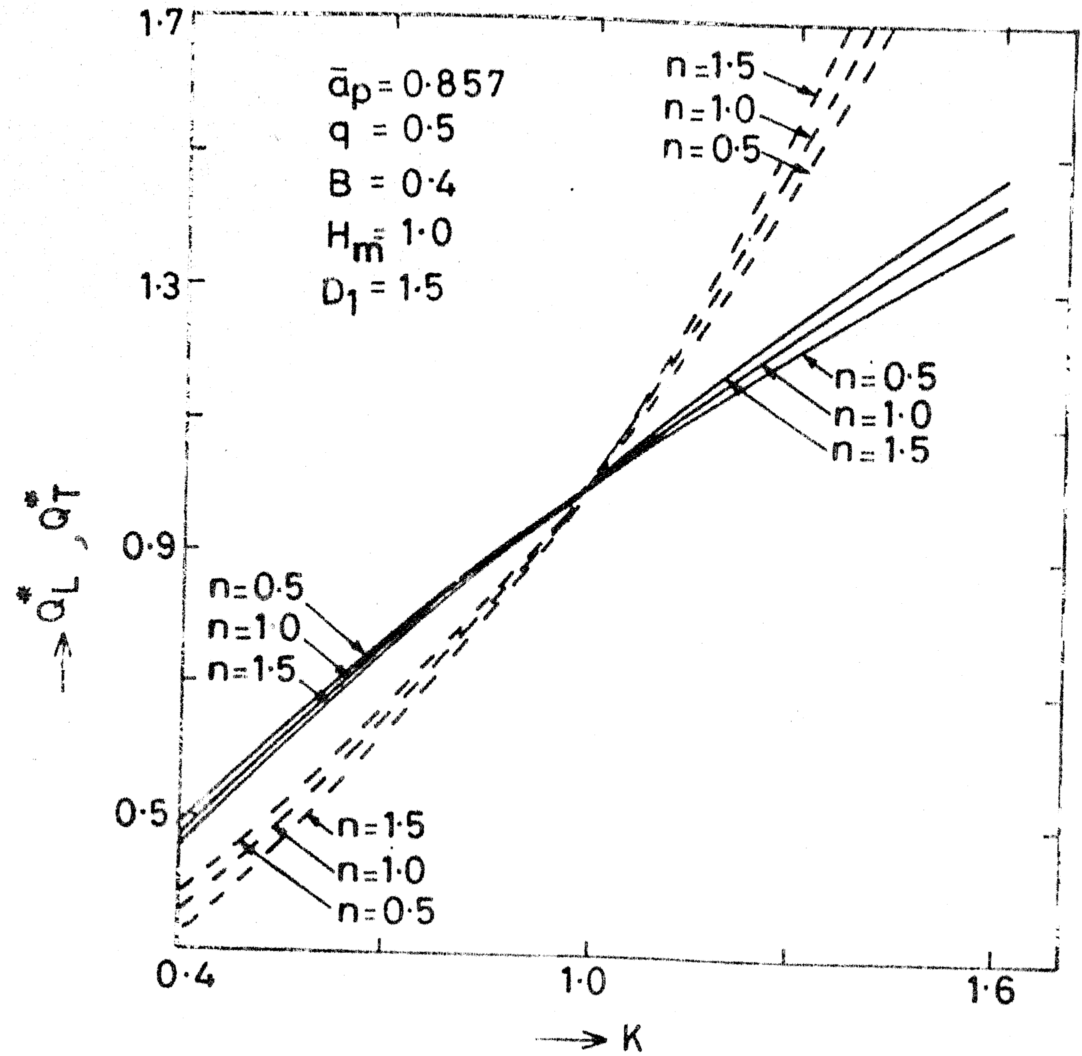

Fig. 7·5 Effect of consistency ratio K on pressure ratio factors Q_L^* & Q_T^* for various values of flow behaviour index n, $H_m \leq 2\,\bar{a}_p$.

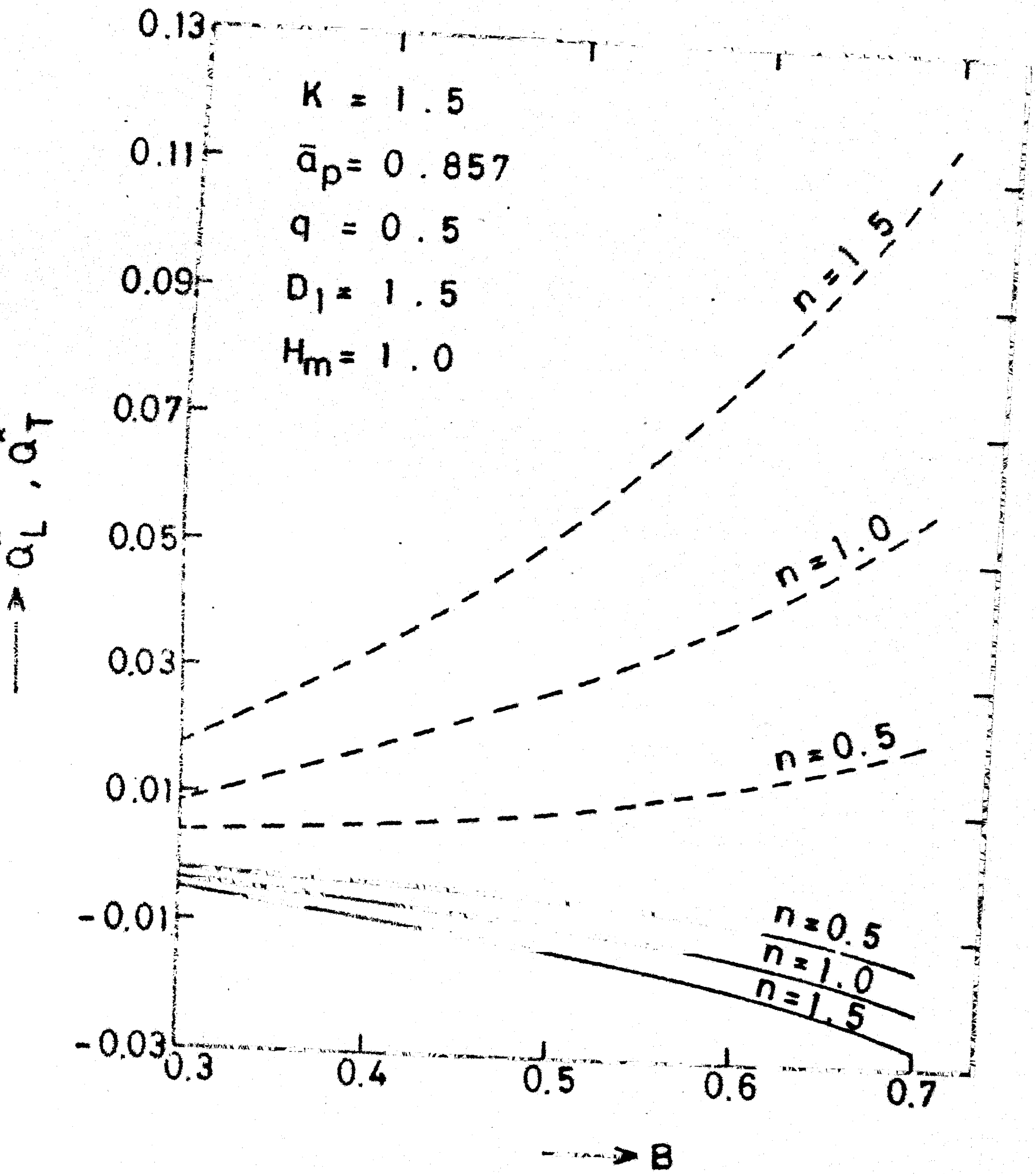

Fig.7.6 Effect of roughness parameter B on pressure ratio factors Q_L^* and Q_T^* for various values of flow behaviour index n, $H_m \leq 2\bar{a}_p$.

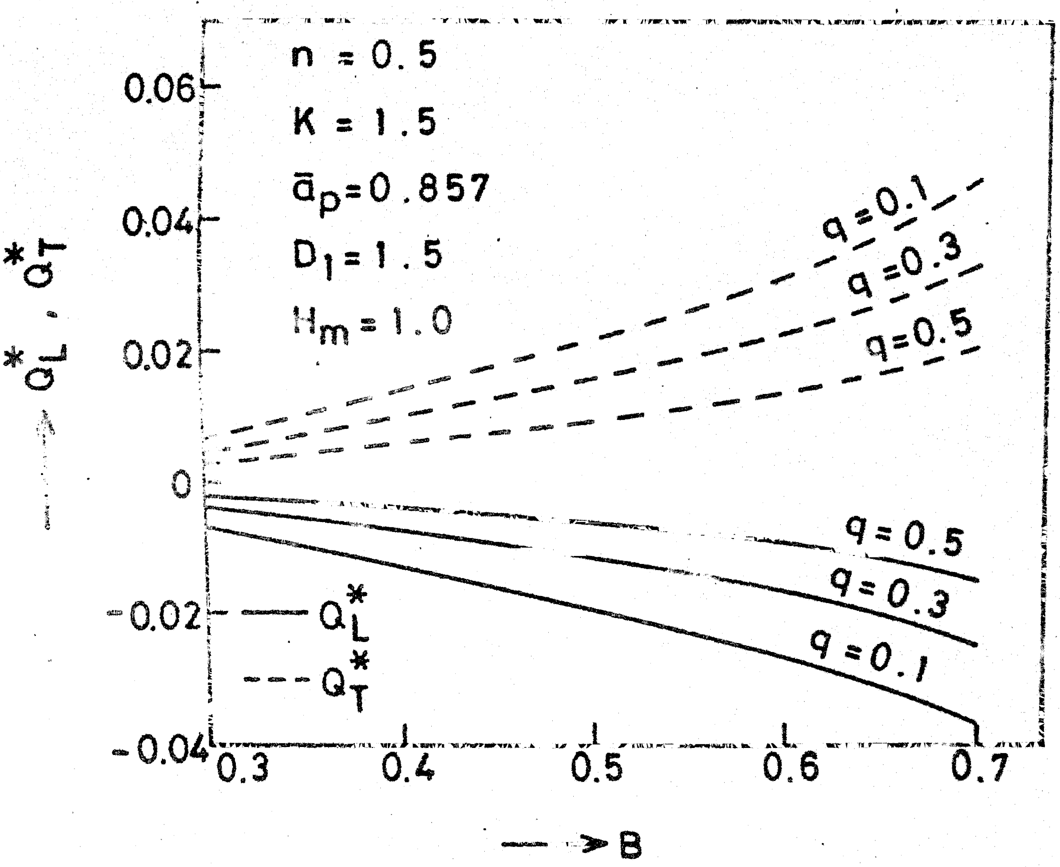

Fig. 7.7 Effect of roughness parameter B on pressure ratio factors Q_L^* and Q_T^* for various values of thermal factor q, $H_m \leqslant 2\,\bar{a}_p$.

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NOMENCLATURE

a	peripheral layer thickness
$\bar{a}$	non-dimensional quantity corresponding to a
d	roughness amplitude measured from the average film thickness
$E(\ )$	expectation of ( )
h	film thickness
h_1	inlet film thickness
h_o	minimum film thickness
h_n	nominal film thickness
h_s	roughness part of stochastic film thickness
H, H_1, H_o, Hn, Hs	non-dimensional quantities corresponding to h, h_1, h_o, h_n and h_s resply
K	consistency ratio
d	half of hertizian width
d_1	the point at which $h_n = 2a$
D_1	ratio = d/d_1
m, m_o, m_1	consistency indices
n	flow behaviour index
p_E	elastohydrodynamic pressure
q^*, q_1^*, q_2^*	reduced pressures
Q_L^*, Q_T^*	pressure ratio factors